

MATHEMATICAL MODELLING AND SIMULATION OF CONTINUOUS FLOTATION CELL / BANK

By

PADMANABH N. MANDLEKAR

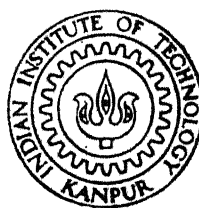
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DEPARTMENT OF METALLURGICAL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY KANPUR

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MATHEMATICAL MODELLING AND SIMULATION OF CONTINUOUS FLOTATION CELL / BANK

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PADMANABH N. MANDLEKAR

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CERTIFICATE

This is to certify that the work entitled "MATHEMATICAL MODELLING AND SIMULATION OF CONTINUOUS FLOTATION CELL/BANK", by Padmanabh N. Mandlekar, has been carried out under my supervision and that this work has not been submitted elsewhere for a degree.



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March, 1991

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Padmanabh N. Mandlekar

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LIST OF SYMBOLS

$M_p(k, t)dk$	Mass of particles in the pulp with flotation rate constant between $k/k+dk$ at time t
$M_f(k, t)dk$	Mass of particles in the froth with flotation rate constant between $k/k+dk$ at time t
$M_c(k, t)dk$	Mass of particles in the concentrate with flotation rate constant between $k/k+dk$ at time t
$M_T(k, t)dk$	Mass of particles in the tails with flotation rate constant between $k/k+dk$ at time t
$\dot{F}(k)dk$	Mass flow rate particles in the feed with flotation rate constant between $k/k+dk$
h	First order flotation rate constant for mass transfer from froth to pulp
t	First order flotation rate constant for mass transfer from froth to concentrate
T	Reciprocal of residence time of particles in the cell
μ	Absolute moment of distribution in pulp
ν	Absolute moment of distribution in froth
η	Absolute moment of distribution in concentrate
z	Absolute moment of distribution in feed for bank or cells
e	Absolute moment of distribution in feed of a single cell
a, b_0, b_1, b_2	Pearson's distribution parameters

Superscripts

j, m Index of a cell in a bank

Subscripts

p Pulp
 c Concentrate

f	Froth
T	Tails
l	Lower limit
u	Upper limit
n	Index of a moment.

ABSTRACT

A multiphase continuously distributed species model of flotation cell has been put forth and steady state kinetics of the flotation process are discussed. For a realistic description of the process, the instantaneous distributions of particles are converted into absolute general moments. A continuous flotation cell and bank of cells is simulated effectively with the help of Pearson's closure technique. The Pearson's suite of moment equations is used to solve the incomplete set of moment equations of the flotation model. For demonstration of method, feed of the particles to the flotation cell/bank in first order flotation rate constant (K) has been characterised in four probabilistic distributions: (1) Uniform, (2) Beta, (3) Gamma and (4) Mixed feeds.

Holdups were computed analytical by integration, except when feed was gamma distribution. The Pearson's value of holdups are in good agreement with the actual holdups. It is concluded that this could be a powerful and versatile method for modelling flotation circuits.

CHAPTER 1

INTRODUCTION

Froth flotation is the commercially most important and technologically most interesting mineral dressing process. With decreasing grade of ores and increased dissemination of target minerals within the ore, the cost and energy consumption in froth flotation has become increasingly important. The incentive for increasing the recovery by as little as say 0.5% is considerable when flotation plants handle large tonnages of ore. To increase production and decrease cost and energy consumption, design and control strategies have been applied to flotation circuits in an empirical manner because no complete realistic analysis of flotation circuits exists. Hence research efforts in flotation must be aimed at improving overall performance of plants. Optimum circuits must be designed and synthesized by developing mathematical strategies and simulating them with computer.

The basic unit of a continuous flotation circuit is the flotation cell. A flotation cell is assumed to be a perfect mixer. A continuous flotation cell splits the incoming stream of particulate solids into a concentrate stream, which is richer in mineral, and a tailing stream which is richer in gangue. Number of flotation cells connected in series constitute a flotation bank. A flotation bank may be referred as rougher cleaner or scavenger depending upon the function it

performs and the grade of the feed to it. A number of banks connected together constitute a flotation circuit.

The primary attributes of particulate ore material are those which are internal or inherent to the feed e.g. particle size, composition, morphology, surface active sites etc. These attributes interact with variables of the flotation process to generate derived attributes. The main process variables are chemical conditioning, pulp density, aeration rate, size and residence time distribution of bubbles, intensity of pulp agitation etc. The derived attributes are hydrophobicity, residence time distribution and of course flotation rate constant, K . In reality flotation cell comprises of many phases which participate in mineral transport and exchange. The physical-chemical phenomenon and the interaction between material and bubble is only partially understood. The flotation process comprises of many micro-events such as particle bubble collision, thin film rupture, particle attachment to bubble and its detachment, levitation of bubble particle aggregate and incorporation in the froth phase. For modelling purpose it is not necessary, nor it is possible to include all elementary rate processes explicitly in the model. Various rate parameters are represented by an apparent first order flotation rate constant, under free flotation conditions.

In the present work a mathematical model has been developed which deals with the steady state kinetics of the flotation process. The model is based on a novel technique for

solving the set of moment equations, that arise in model formulation, by means of Pearson's suite of moment equations. Before proceeding with present work, a detail survey of existing literature is presented. It will enable the reader to get a good insight into the work.

CHAPTER 2

LITERATURE SURVEY

We first consider the kinetics of flotation, beginning with simple models of semi-batch flotation in a single cell.

Note: All references in this chapter are taken from ref.3

2.1. SEMI-BATCH FLOTATION KINETICS :

A large number of mathematical models have been proposed for the semi-batch system. Lynch et al. (1981) have categorised these models into empirical, probabilistic and kinetic models. The kinetic models are best classified in a P-S format (Figure 2.1). Each class of model lies at the intersection of phase P and feed species S, where P represents a single or multiphase system, and S can be either a single class of species or discretely distributed or continuously distributed ensemble of particles. Six types of models evident from the figure are explained below.

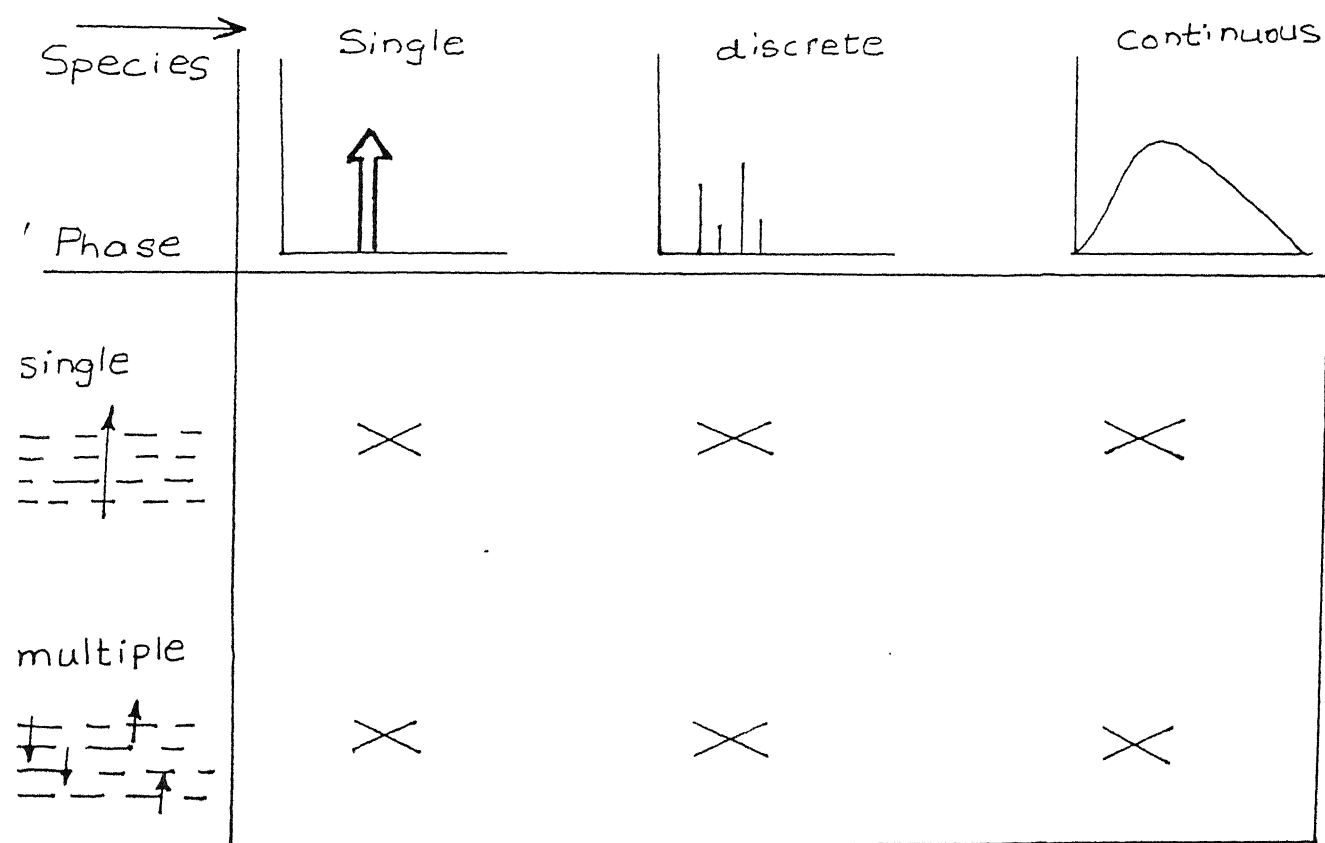
2.1.1. SINGLE PHASE - SINGLE SPECIES MODELS :

Garcia-Zungia (1935) first recognized flotation as a rate process and a first order rate of flotation of single species from pulp phase.

$$\frac{d M_p(t)}{dt} = - K M_p(t) \quad (2.1)$$

where $M_p(t)$ is mass of particle remaining in pulp at flotation time t . As this model is not adequate, chemical reaction

Fig 2.1 P-S MATRIX CLASSIFICATION OF
FLOTATION KINETIC MODELS



kinetics analogy has been used for suggesting second order (Arbiter, 1951) and higher order (Debruyn and Modi, 1956; Volin and Swami, 1965) flotation models. The nth order model is

$$\frac{d M_p(t)}{dt} = - K M_p^n(t) \quad (2.2)$$

Direct entrainment of solids from pulp to froth is a secondary path of material transport. In the case of hydrophilic (and especially fine) particles significant material transport (Thorne et al., 1976) occurs by the secondary path. Consequently possible modification of the first order model could be

$$\frac{d M_p(t)}{dt} = - K M_p(t) - J \quad (2.3)$$

where J is the entrainment rate.

2.1.2. SINGLE PHASE - DISCRETELY DISTRIBUTED SPECIES MODEL :

In a logical extension to the first order kinetics model, the feed is divided into X classes of homogeneous species, wherein all members of a class possess identical rate constant value (Imaizumi and Inoue, 1965). Presence of two classes is implicit in the kinetic model of Morris (1952) who introduced an additional parameter to account for the empirical fact that the flotation may or may not proceed to completion.

$$\frac{d M_p(t)}{dt} = - K [M_p(t) - M_p(\infty)] \quad (2.4)$$

where $M_p(\infty)$ is the non floating material.

2.1.3. SINGLE PHASE - CONTINUOUSLY DISTRIBUTED SPECIES MODELS :

In the case of heterogeneous ground ores with incomplete liberation the number of discrete classes become large and the feed is best modelled as continuously distributed in rate constants.

$$M_p(t) = \int_0^{\infty} M_p(K, 0) \exp(-Kt) dK \quad (2.5)$$

where $M(K, 0)dK$ is mass or mass fraction of original feed with flotation rate constant of K to $K + dK$. Hence $M(K, 0)$ is a relative or absolute density function, and

$$\begin{aligned} \int_0^{\infty} M_p(K, 0) dK &= 1 \quad \text{or} \\ &= M_p(0) \end{aligned} \quad (2.5a)$$

where $M_p(0)$ is the total feed to the cell. Analytical expressions exist only for a few distributions. The best known example is gamma distribution in parameters u and n (Woodburn and Loveday, 1965; Loveday, 1966; Inoue and Imaizumi, 1968; Harris and Chakravarti, 1970)

$$M_p(K, 0) = \frac{u^n}{\Gamma(n)} K^{n-1} \exp(-uK) \quad (2.6)$$

Equation (2.6) on substitution in (2.5) yields

$$M_p(t) = \left[\frac{u}{u + t} \right]^n \quad (2.7)$$

A bimodal distribution generated by mixing two gamma distributions in the ratio ϕ and $(1 - \phi)$ can yield a more versatile model (Harris and Chakravarti, 1970)

$$M_p(t) = (1 - \emptyset) \left[\frac{u_1}{u_1 + t} \right]^{n_1} + \emptyset \left[\frac{u_2}{u_2 + t} \right]^{n_2} \quad (2.8)$$

In case of rectangular distribution in the range of $K_1 < K < K_u$ the model is (Huber-Panu et al., 1976)

$$M_p(t) = \frac{\exp(-K_1 t) - \exp(-K_u t)}{(K_u - K_1)t} \quad (2.9)$$

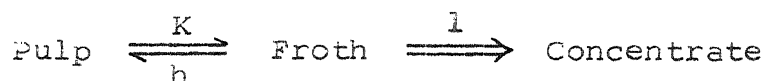
A particular statistical distribution function e.g. beta, gamma or uniform is chosen as a matter of convenience for extracting close form mathematical expressions for $M_p(t)$ or $M_c(t)$. Harris and Chakravarti (1970) recognized that $M_p(t)$ is the laplace transform of $R_p(K, 0)$. Kapur and Mehrotra (1974) devised a reliable and robust technique for computing the feed distribution from flotation data. Their scheme is based on the fact that flotation kinetics can be always expressed in cumulative distribution function $R_p(K, 0)$ as follows.

$$\bar{M}_p(t) = \frac{1 - M_p(t)}{t} = \int_0^\infty R_p(K, 0) \exp(-Kt) dK \quad (2.10)'$$

2.1.4. MULTIPHASE - SINGLE SPECIES MODEL :

In reality the flotation comprises of many phases which participate in material transport and exchange, therefore increasing only the complexity of single phase kinetic models may not be productive. None of the parameters in the multiphase flotation systems can be computed from first principles, but must be estimated from experimental data with great deal of uncertainty regarding the reliability and significance of

estimate. A simple model of multiphase species was suggested by Harris and Rimmer (1966)



The governing equations of this model in pulp, froth and concentrate phases are respectively.

$$\frac{d M_p(t)}{dt} = -K M_p(t) + h M_f(t) \quad (2.11)$$

$$\frac{d M_f(t)}{dt} = K M_p(t) - h M_f(t) - 1 M_f(t) \quad (2.12)$$

$$\frac{d M_c(t)}{dt} = 1 M_f(t) \quad (2.13)$$

with the initial conditions $M_f(0) = M_c(0) = 0$. Simultaneous solution of these equations yields recovery.

$$M_c(t) = M_c(\infty) \left(1 - \frac{1}{(A-B)} [A \exp(Bt) - B \exp(At)] \right) \quad (2.14)$$

where A and B are

$$A = - (k + h + 1)/2 - \sqrt{(h + k + 1)^2 - 4k l/2} \quad (2.15a)$$

$$B = - (k + h + 1)/2 + \sqrt{(h + k + 1)^2 - 4k l/2} \quad (2.15b)$$

Under highly mineralized conditions, if the rate of drainage becomes negligible this model simplifies to

$$M_c(t) = M_c(\infty) \left(1 - \frac{1}{(1-k)} [1 \exp(-kt) - k \exp(-lt)] \right) \quad (2.16)$$

2.1.5. MULTIPHASE DISCRETELY DISTRIBUTED SPECIES MODELS :

Equation (2.14) can be applied to more than one class of species. It merely requires summing this expression over all classes.

2.1.6. MULTIPHASE - CONTINUOUSLY DISTRIBUTED SPECIES MODELS :

This category of models provide the most general and realistic description of flotation kinetics. The main difficulty in implementing these models lies in Equation (2.14) which after combining with the feed distribution in k cannot be integrated in a close form. Ball et al. (1970) got around this problem to some extent by converting the instantaneous distributions of particles in pulp, froth, and concentrate phases into time dependent absolute general moments. The n th moments in pulp, froth and concentrate are defined as

$$\mu_n(t) = \int_0^{\infty} M_p(k, t) k^n dk \quad (2.17)$$

$$\nu_n(t) = \int_0^{\infty} M_f(k, t) k^n dk \quad (2.18)$$

$$\eta_n(t) = \int_0^{\infty} M_c(k, t) k^n dk \quad (2.19)$$

Multiplying throughout with k^n and integrating over the range of k , Eqs. (2.11) to (2.13) are readily transformed into

$$\frac{\int_0^{\infty} d M_p(k, t) k^n dk}{dt} = - \int_0^{\infty} k M_p(k, t) k^n dk + h \int_0^{\infty} M_f(k, t) k^n dk \quad (2.20)$$

$$\frac{\int_0^{\infty} \frac{d}{dt} M_f(k, t) k^n dk}{dt} = \int_0^{\infty} k M_p(k, t) k^n dk - h \int_0^{\infty} M_f(k, t) k^n dk - 1 \int_0^{\infty} M_f(k, t) k^n dk \quad (2.21)$$

$$\frac{\int_0^{\infty} \frac{d}{dt} M_c(k, t) k^n dk}{dt} = 1 \int_0^{\infty} M_f(k, t) k^n dk \quad (2.22)$$

Inserting equations (2.17) to (2.19) yields:

$$\frac{d u_n(t)}{dt} = - u_{n+1}(t) + h v_n(t) \quad (2.23)$$

$$\frac{d v_n(t)}{dt} = u_{n+1}(t) - (h + 1) v_n(t) \quad (2.24)$$

$$\frac{d \eta_n(t)}{dt} = 1 v_n(t) \quad (2.25)$$

This incomplete set of equations was closed with the help of La Guerre series approximation which provides an additional expression for simultaneous solution of the above set (B. Ball, P.C. Kapur and D.W. Fuerstenau, 1970). This closure unfortunately is quite crude and does not provide reasonable accurate solution.

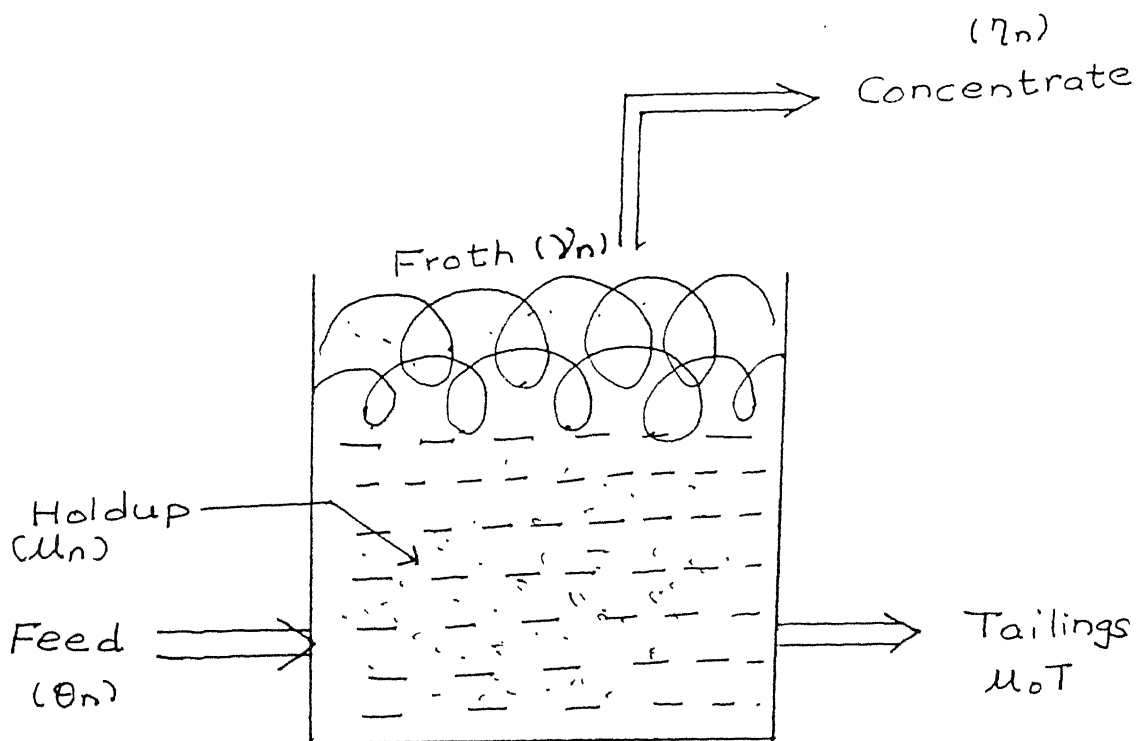
2.2. SIMULATION OF FLOTATION CIRCUITS :

Any scheme for simulation of flotation circuits has three main components feed specifications, flotation model and split factor, and circuit configuration.

2.2.1. FEED SPECIFICATIONS :

Feed can be described as a density function of three

Fig 2.2 SCHEMATIC OF FLOTATION CELL



attributes, K first order flotation rate constant, $K_L < K < K_U$ and $K_L > 0$ and $K_U < \infty$; L the particle size, and V the volume fraction of target mineral, $0 \leq v \leq 1$. But continuous trivariate distribution is neither easy to identify nor amenable to required manipulations in circuit modelling. Therefore we focus our attention on feed in single attribute, K first order flotation constant. Feeds are described by continuous distributions such as uniform, beta, gamma etc. Continuous distributions may be descretised into five or more intervals as error introduced is very little. Continuous distributions are susceptible to more elegant mathematical treatment than are discrete distributions. The essential property of a function of continuous random variable is that area under this curve is always equal to one.

2.3. MODEL OF A CONTINUOUS CELL :

A continuous flotation cell is one to which feed is continuously fed and concentrate and tailings are continuously drawn out (refer Figure 2.2). The cell is assumed to be a perfect mixer. Let h be first order rate constant for mass transfer from froth to pulp, l first order rate constant for mass transfer from froth to concentrate, $T = 1/t$, t the residence time of particles in the cell. Steady state mass balance equations for particles having flotation constant between $k/k + dk$ are

$$\hat{F}(k) - k M_p(k) + h M_f(k) - T M_p(k) = 0 \quad (2.26)$$

Also

$$k M_p(k) - h M_f(k) - l M_{\bar{f}}(k) = 0 \quad (2.27)$$

$$\dot{M}_c(k) = l M_{\bar{f}}(k) \quad (2.28)$$

Rearranging the terms

$$M_{\bar{f}}(k) = \frac{k}{(h + l)} M_p(k) \quad (2.29)$$

Substituting Eq. (2.29) in Eq. (2.26)

$$\dot{F}(k) = \left(\frac{k l}{(h + l)} + T \right) M_p(k) \quad (2.30)$$

i.e.

$$\dot{F}(k) = \frac{(hT + lT + kl)}{(h + l)} M_p(k) \quad (2.31)$$

$$\dot{M}_c(k) = \frac{k l}{(h + l)} M_p(k) \quad (2.32)$$

$$\dot{M}_T(k) = T M_p(k) \quad (2.33)$$

Equations (2.31), (2.32) and (2.33) are the model equations of a continuous cell. The continuous cell is simulated by integrating $M_c(k)$ over all k

$$M_c(k) = \frac{kl}{(hT + lT + kl)} F(k) \quad (2.34)$$

$$M_c = \int_0^{\infty} M_c(k) dk = \int_0^{\infty} \frac{kl}{(hT + lT + kl)} F(k) dk \quad (2.35)$$

This format of the model is not convenient for multi-cell, multi-bank circuit. Our objective is to reformulate the model in differential equations in moments.

CHAPTER 3

OBJECTIVES OF PRESENT STUDY

The present study aims at developing models for continuous flotation kinetics of a single cell and bank of cells respectively. The model formulation consists of an incomplete set of moment equations presented in the last chapter. We demonstrate a new technique for solving the incomplete set of moment equations by means of Pearson's suite of moment equations. For verification of our method the computed results are compared with analytical results wherever available.

CHAPTER 4

FLOTATION MODEL IN MOMENTS DOMAIN

4.1. SIMULATION OF A FLOTATION CELL :

The model equations governing the transfer of the differential element of material, with rate constants between $k/k+dk$ in a continuous single cell at steady state are

$$\dot{M}_C(k) = \frac{1}{(h+1)} \frac{k}{k+dk} M_P(k) \quad (4.1)$$

$$\dot{F}(k) = \left(\frac{k}{(h+1)} + T \right) M_P(k) \quad (4.2)$$

$$\dot{M}_T(k) = T M_P(k) \quad (4.3)$$

Since we are interested in total mass in each phase the following moments are defined. The moments of continuous distribution give more realistic description of the process than the descretized distributions.

$$\Theta_n = \int_0^{\infty} F(k) k^n dk \quad (4.4)$$

$$\mu_n = \int_0^{\infty} M_P(k) k^n dk \quad (4.5)$$

$$\eta_n = \int_0^{\infty} M_C(k) k^n dk \quad (4.6)$$

where

Θ_0 is total mass of feed;

μ_0 is total mass of pulp i.e. holdup and

η_0 is total mass of concentrate.

Converting equations (4.1), (4.2) and (4.3) into moment equations

$$\int_0^{\infty} \dot{M}_c(k) k^n dk = \int_0^{\infty} \frac{1-k}{(h+1)} M_p(k) k^n dk \quad (4.7)$$

$$\int_0^{\infty} \dot{F}(k) k^n dk = \int_0^{\infty} \frac{1-k}{(h+1)} M_p(k) k^n dk + T \int_0^{\infty} M_p(k) k^n dk \quad (4.8)$$

$$\int_0^{\infty} \dot{M}_T(k) k^n dk = T \int_0^{\infty} M_p(k) k^n dk \quad (4.9)$$

$$\dot{\theta}_n = 1/(h+1) \mu_{n+1} \quad (4.10)$$

$$\dot{\theta}_n = 1/(h+1) \mu_{n+1} + T \mu_n \quad (4.11)$$

$$\dot{z}_n = T \mu_n \quad (4.12)$$

Solving (4.10), (4.11) and (4.12) yields the desired information about feed, pulp, concentrate in moments form. In (4.11) h , l , T and θ_n are known variables of the flotation process. To solve the equation, we put $n = 0, 1, 2, \dots$ etc.

$$\dot{\theta}_0 = 1/(h+1) \mu_1 + T \mu_0 \quad (4.13a)$$

$$\dot{\theta}_1 = 1/(h+1) \mu_2 + T \mu_1 \quad (4.13b)$$

$$\dot{\theta}_2 = 1/(h+1) \mu_3 + T \mu_2 \quad (4.13c)$$

Thus there is always one more unknown than the number of equations to be solved. One more equation is required for obtaining a unique solution. This problem of closure of incomplete set of equations can be tackled in 2 ways:

- (1) La-Guerre's approach and
- (2) Pearson's approach.

4.2. LA-GUERRE'S APPROACH :

La-Guerre's series approximation gives an approximate relationship between moments of a distribution function (B. Ball, P.C. Kapur and D.W. Fuerstenau, 1970)

$$\mu_3 = \frac{\mu_2}{\mu_0 \mu_1} (2\mu_0 \mu_2 - \mu_1^2) \quad (4.14)$$

Equations (4.13a), (4.13b), (4.13c) and (4.14) can be solved numerically on a computer. This closure was however found to be inaccurate.

4.3. PEARSON'S CLOSURE APPROACH :

Pearson's closure provides an exceptionally flexible relationship between various moments of a function. This relation is used to close a set of incomplete moment equations.

4.3.1. PEARSONIAN DISTRIBUTION :

Failure of normal distributions to fit many of distributions encountered in real life necessitated development of a generalised system of probability distributions. Karl Pearson put forth a distribution which can represent almost all statistical distributions used in practice (Reference 1).

$$\frac{df}{dx} = \frac{(x - a)f}{(b_0 + b_1x + b_2x^2)} \quad (4.15)$$

where $f(x)$ is the density function and a, b_0, b_1, b_2 are disposable parameters of the distributions. It is not necessary to take terms beyond b_2x^2 . Depending on Pearson's constants $a,$

b_0 , b_1 and b_2 , the above differential equation can give curve of various shapes. Pearsonian distribution may be converted into moment form as follows. Multiplying both sides by x^n and integrating from 0 to $+\infty$ by parts

$$\int_0^{\infty} x^n (b_0 + b_1 x + b_2 x^2) \frac{df}{dx} dx = \int_0^{\infty} (x - a) x^n f dx \quad (4.16)$$

$$\begin{aligned} & [(b_0 + b_1 x + b_2 x^2) x^n f]_0^{\infty} - \int_0^{\infty} (nb_0 x^{n-1} + (n+1)b_1 x^n \\ & + (n+2)b_2 x^{n+1}) f dx = \int_0^{\infty} (x - a) x^n f dx \end{aligned} \quad (4.17)$$

Assuming higher order contact at extremities terms in square brackets tend to zero. On simplification we have

$$-a\mu_n + nb_0\mu_{n-1} + (n+1)b_1\mu_n + (n+2)b_2\mu_{n+1} + \mu_{n+1} = 0 \quad (4.18)$$

4.3.2. MOMENT EQUATIONS :

Pearson's suite of moment equations are obtained by putting $n = 0$ to 4. Since there are only 4 Pearson's constants to be determined, we put $n = 0$ to 3. The fifth equation obtained by putting $n = 4$ verifies the Pearson's fit.

$n = 0$:

$$-a\mu_0 + 0 + b_1\mu_0 + b_2(2\mu_1) + \mu_1 = 0 \quad (4.19)$$

$n = 1$:

$$-a\mu_1 + b_0\mu_0 + b_1(2\mu_1) + b_2(3\mu_2) + \mu_2 = 0 \quad (4.20)$$

$n = 2$:

$$-a\mu_2 + b_0(2\mu_1) + b_1(3\mu_2) + b_2(4\mu_3) + \mu_3 = 0 \quad (4.21)$$

$n = 3:$

$$-a\mu_3 + b_0(3\mu_2) + b_1(4\mu_3) + b_2(5\mu_4) + \mu_4 = 0 \quad (4.22)$$

$n = 4:$

$$-a\mu_4 + b_0(4\mu_3) + b_1(5\mu_4) + b_2(6\mu_5) + \mu_5 = 0 \quad (4.23)$$

Equations (4.19) to (4.22) form a set of non linear algebraic equations. (4.23) is a check equation. Correspondingly, we now put $n = 0$ to 4 in the model equation (4.11).

$n = 0:$

$$\dot{\theta}_0 - (1/(h+1))\mu_1 - T\mu_0 = 0 \quad (4.24)$$

$n = 1:$

$$\dot{\theta}_1 - (1/(h+1))\mu_2 - T\mu_1 = 0 \quad (4.25)$$

$n = 2:$

$$\dot{\theta}_2 - (1/(h+1))\mu_3 - T\mu_2 = 0 \quad (4.26)$$

$n = 3:$

$$\dot{\theta}_3 - (1/(h+1))\mu_4 - T\mu_3 = 0 \quad (4.27)$$

$n = 4:$

$$\dot{\theta}_4 - (1/(h+1))\mu_5 - T\mu_4 = 0 \quad (4.28)$$

Now, we have 10 equations (4.19) to (4.23) and (4.24) to (4.28) and 10 variables μ_0 to μ_5 and a, b_0, b_1, b_2 . These equations can be readily solved on computer.

4.3.3. SOLUTION OF EQUATIONS :

The algorithm employed is basically a single variable search on μ_0 that satisfies the check equation. This procedure

is employed to exploit the characteristic structure of this problem.

- (1) Read h, l, T .
- (2) Read feed moments θ_0 to θ_5 .
- (3) Make initial guess of holdup (μ_0).
- (4) Determine μ_1 to μ_5 using model equations (4.24) to (4.28).
- (5) If variance of the normalised holdup distribution is negative then go to 3.

$$\text{Holdup variance} = \frac{\mu_2}{\mu_0} - \left(\frac{\mu_1}{\mu_0}\right)^2.$$

- (6) Solve (4.19) to (4.22) by Gauss's elimination.
- (7) Substitute values of all variables in (4.23).
- (8) If (4.23) satisfied then μ_0 is the answer

else

begin

$$\mu_0 = \mu_0 \pm \delta$$

Repeat steps 4 to 8

end.

$$(9) \quad \dot{\eta}_0 = \frac{1}{(h+1)} \mu_1$$

$$(10) \quad \dot{z}_0 = T \mu_0$$

(11) End.

The correct μ_0 is found by bisection method after bracketing the root. The search on μ_0 simplifies the task immensely.

Once μ_0 is known, μ_1 to μ_5 are found from the model equations (4.24) to (4.28). Equations (4.19) to (4.22) are solved by

Gauss's elimination as 4 equations in 4 unknowns a, b_0, b_1, b_2 . Equation (4.23) verifies the Pearson's fit. In this way, a flotation cell is simulated using Pearson's closure.

4.4. MODEL EQUATIONS FOR A BANK OF CELLS :

Consider a flotation bank with m cells in series. Let $j = 2, 3, \dots, m$. For the first cell the model equations are

$$\dot{M}_C^1(k) = \frac{1}{(h+1)} \frac{k}{1} M_P^1(k) \quad (4.29)$$

$$\dot{F}^1(k) = \left(\frac{k}{(h+1)} + T \right) M_P^1(k) \quad (4.30)$$

$$\dot{M}_T^1(k) = T M_P^1(k) \quad (4.31)$$

Model equations in moment form are obtained by multiplying both sides by k^n and then integrating both sides from 0 to $+\infty$

$$\dot{\eta}_n^1 = 1/(h+1) \mu_{n+1}^1 \quad (4.32)$$

$$\dot{\theta}_n^1 = 1/(h+1) \mu_{n+1}^1 + T \mu_n \quad (4.33)$$

$$\dot{z}_n^1 = T \mu_n \quad (4.34)$$

For $j = 2$ cell the model equations are

$$\dot{M}_T^1(k) = \left(\frac{k}{(h+1)} + T \right) M_P^2(k) \quad (4.35)$$

$$\dot{M}_C^2(k) = \frac{k}{(h+1)} \frac{1}{1} M_P^2(k) \quad (4.36)$$

$$\dot{M}_T^2(k) = T M_P^2(k) \quad (4.37)$$

Converting the model equations into moment form, we have

$$\dot{\eta}_n^2 = 1/(h+1)\mu_{n+1}^2 \quad (4.38)$$

$$\dot{z}_n^1 = 1/(h+1)\mu_{n+1}^2 + T \mu_n^2 \quad (4.39)$$

$$\dot{z}_n^2 = T \mu_n^2 \quad (4.40)$$

For any j , $j = 2, 3, \dots, m$

$$\dot{\eta}_n^j = 1/(h+1)\mu_{n+1}^j \quad (4.41)$$

$$\dot{z}_n^{j-1} = 1/(h+1)\mu_{n+1}^j + T \mu_n^j \quad (4.42)$$

$$\dot{z}_n^j = T \mu_n^j \quad (4.43)$$

For the last cell the model equations are

$$\dot{M}_T^{m-1}(k) = \left(\frac{k+1}{(h+1)} + T \right) M_P^m(k) \quad (4.44)$$

$$\dot{M}_C^m(k) = \frac{k+1}{(h+1)} M_P^m(k) \quad (4.45)$$

$$\dot{M}_T^m(k) = T M_P^m(k) \quad (4.46)$$

Converting the model equations into moment form, we have

$$\dot{\eta}_n^m = 1/(h+1)\mu_n^m \quad (4.47)$$

$$\dot{z}_n^{m-1} = 1/(h+1)\mu_{n+1}^m + T \mu_n^m \quad (4.48)$$

$$\dot{z}_n^m = T \mu_n^m \quad (4.49)$$

The model equations of bank or cells are solved using the same algorithm, used for a single cell. Total concentrate of the bank is

$$\dot{m}_C = \sum_{j=1}^m \dot{m}_O^j = \frac{1}{(n+1)} \sum_{j=1}^m \mu_1^j \quad (4.50)$$

Total tailing flow rate out of the bank is

$$\dot{m}_T = T \mu_O^m \quad (4.51)$$

CHAPTER 5

RESULTS

The theory and procedure of solving the model equations have been discussed in Chapter 4. In this chapter verification of technique is demonstrated by using those distribution functions for which results can be derived analytically. Hence can be compared with Pearson's closure method.

Results consist of 2 parts; those pertaining to flotation

- (1) Single flotation cell
- (2) Bank of flotation cells.

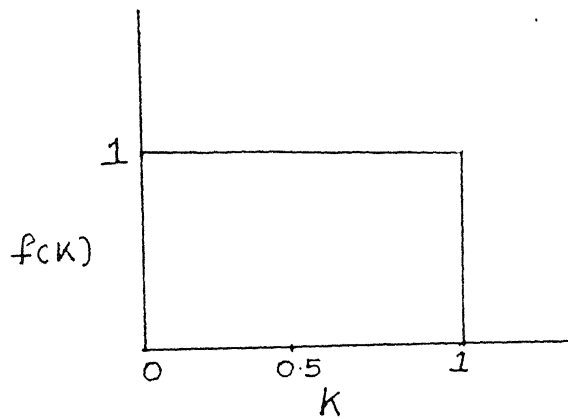
5.1. SINGLE FLOTATION CELL

Feed to the single flotation cell is characterised by 4 statistical distributions:

- (1) Uniform distribution
- (2) Gamma distribution
- (3) Beta distribution
- (4) Mixed distribution of above.

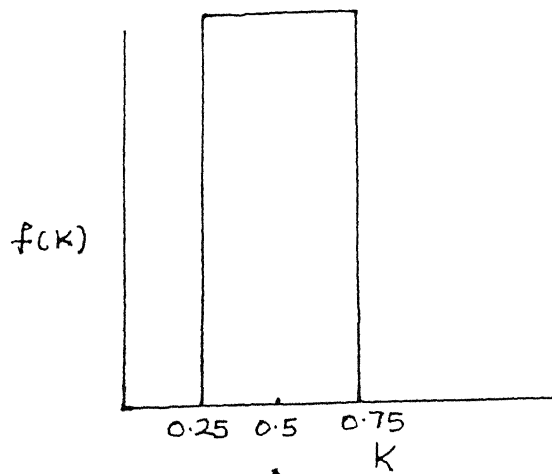
Parameters h , l , T , Θ_n (the moments of feed) are the known parameters for a flotation cell, and μ_n , η_n (the moments of holdup and concentrate) and tailings flows are the unknowns. First, we present analytical expressions for Θ_n and holdup.

Fig 5.12 UNIFORM DISTRIBUTION

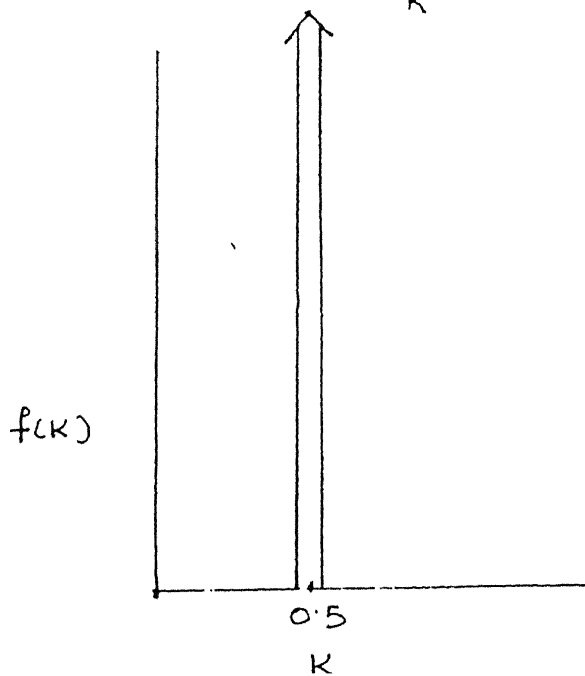


$$f(K) = \frac{1}{K_u - K_l}$$

$$K_l = 0 \quad K_u = 1$$



$$K_l = 0.25 \quad K_u = 0.75$$



$$K_l = 0.49 \quad K_u = 0.51$$

Impulse.

5.1.1. FEED DISTRIBUTION MOMENTS, Θ_n :

(1) Uniform or rectangular distribution

$$f(K) = \frac{100}{K_u - K_l} ; \quad K_l \leq K \leq K_u \quad (5.1)$$

K_l and K_u are the variable parameters of this distribution which yield different distributions (Figure 5.12).

The n th moment is:

$$\begin{aligned} \Theta_n &= \int_{K_l}^{K_u} f(K) K^n dK \\ \Theta_n &= \frac{100}{K_u - K_l} \int_{K_l}^{K_u} K^n dK \end{aligned} \quad (5.2)$$

$$\Theta_n = \frac{100}{K_u - K_l} \times \frac{1}{(n+1)} \times (K_u^{n+1} - K_l^{n+1}) \quad (5.3)$$

$$\Theta_0 = 100 \quad \Theta_1 = 50$$

We consider all uniform distributions with normalised mean \bar{K}

$$= \frac{\Theta_1}{\Theta_0} = \frac{50}{100} = 0.5$$

$$\begin{aligned} \text{e.g. } K_l &= 0 & K_u &= 1 \\ K_l &= 0.1 & K_u &= 0.9 \end{aligned}$$

Dispersion index is defined as

$$\text{Dispersion Index} = \sqrt{\frac{\text{Variance}}{(\text{Mean})^2}} \quad (5.4)$$

$$\text{Dispersion Index} = \sqrt{\frac{\Theta_0 \Theta_2 - \Theta_1^2}{\Theta_1^2}} \quad (5.5)$$

For $K_L = 0$, $K_U = 1$ substitution of θ_0 , θ_1 and θ_2 from equation (5.3) gives $\theta_0 = 100$ $\theta_1 = \frac{100}{2}$ $\theta_2 = \frac{100}{3}$

$$\text{Dispersion Index} = \sqrt{\frac{\left(\frac{100^2}{3}\right) - (50)^2}{(50)^2}} \quad (5.6)$$

$$\text{Dispersion Index} = 0.333$$

It will be seen that dispersion tends to zero as $K_U - K_L$ tends to a point value (i.e. the impulse) or when all particles have identical flotation rate.

(2) Gamma distribution

$$f(K) = 100 \frac{w^p}{\Gamma(p)} K^{p-1} e^{-wK} \quad (5.7)$$

w and p are the variable parameters of this distribution (figure 5.13a)

$$\theta_n = 100 \frac{w^p}{\Gamma(p)} \int_0^\infty K^{n+p-1} e^{-wK} dK \quad (5.8)$$

or

$$\theta_n = 100 \frac{\Gamma(p+n)}{\Gamma(p)} \times \frac{1}{w^n} \quad (5.9)$$

Hence

$$\theta_1 = 100 \frac{p!}{(p-1)!} \times \frac{1}{w} = 100 \frac{p}{w} \quad (5.10)$$

And the normalised mean is

$$\frac{\theta_1}{\theta_0} = \frac{p}{w} \quad (5.11)$$

Again we consider only those gamma distributions which have mean $\bar{K} = 0.5$, hence $w = 2p$, and

$$\theta_n = 100 \frac{\sqrt{p+n}}{\sqrt{p}} \times \frac{1}{(2p)^n} \quad (5.12)$$

(3) Beta distribution

$$f(K) = \frac{100}{\sqrt{\gamma} \sqrt{\eta}} \frac{\sqrt{\gamma+n}}{\sqrt{\eta}} K^{\gamma-1} (1-K)^{\eta-1} \quad (5.13)$$

$$0 \leq K \leq 1$$

γ, η are the variable parameters of the distribution (Figure 5.13b)

$$\theta_n = \frac{100}{\sqrt{\gamma} \sqrt{\eta}} \frac{\sqrt{\gamma+n}}{\sqrt{\eta}} p(\gamma+n, \eta) \quad (5.14)$$

$$\theta_n = 100 \frac{\sqrt{\gamma+n}}{\sqrt{\gamma} \sqrt{\gamma+\eta+n}}$$

$$\theta_0 = 100$$

For $n = 1$

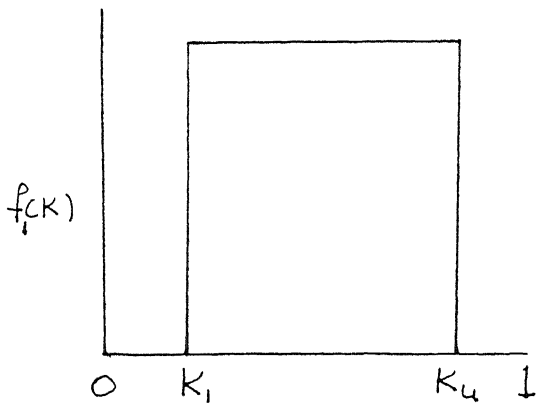
$$\theta_1 = 100 \frac{\gamma}{\gamma+\eta} \quad (5.15)$$

$$\text{Normalised mean } \bar{K} = \frac{\theta_1}{\theta_0} = \frac{\gamma}{\gamma+\eta}$$

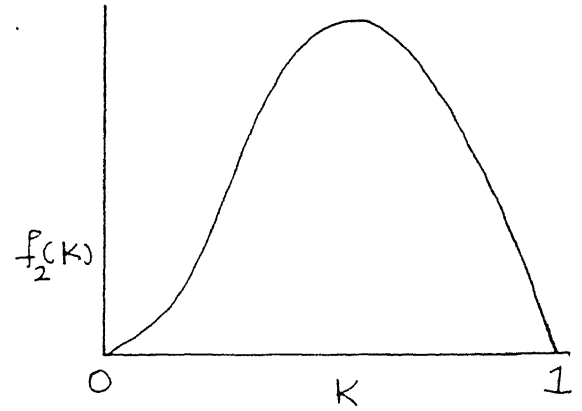
We have considered nine types of beta distributions with different mean \bar{K} , as listed in Table 5.1.

Fig 5.13 C MIXED FEEDS DISTRIBUTION

$$f(k) = m_1 f_1(k) + m_2 f_2(k) \quad [m_1 + m_2 = 1]$$



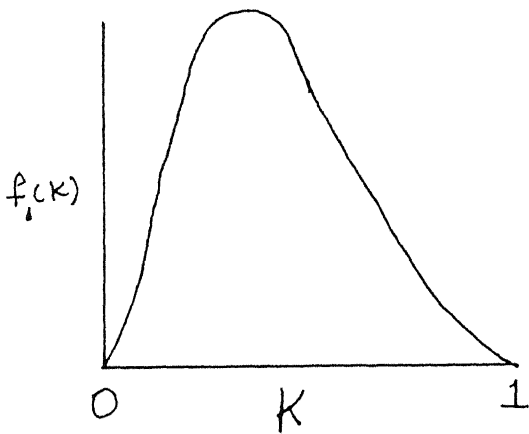
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$$f(k) = m_1 f_1(k) + m_2 f_2(k) \quad [m_1 + m_2 = 1]$$

$$\gamma = 2, \eta = 3$$

$$\gamma = 3, \eta = 2$$



+

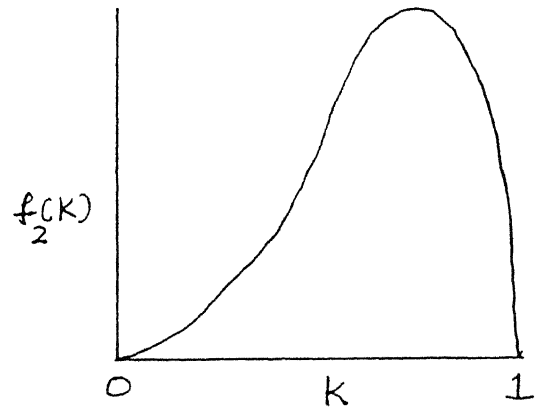


Table 5.1. Beta distributions with different mean \bar{K}

γ	η	mean \bar{K}
1	1	0.50
1	2	0.33
2	1	0.66
1	3	0.25
3	1	0.75
1	4	0.20
4	1	0.80
2	3	0.40
3	2	0.60

(4) Mixed feeds distribution: It is a mixture of two distributions having different mean \bar{K} , in different proportions

$$f(K) = m_1 f_1(K) + m_2 f_2(K) \quad (5.16)$$

$$\text{where } m_1 + m_2 = 1 \quad (5.17)$$

(refer Figure 5.13c).

Two types of mixed feeds distributions are considered.

(1) Mixed feeds of uniform $f_u(K; K_1 = 0, K_u = 1)$ and beta distribution $f_\beta(K; \gamma = 2, \eta = 3)$.

Moments of this mixed feeds distribution are:

$$\begin{aligned} \theta_n = & m_1 \times \frac{100}{(K_u - K_1)} \times \frac{1}{(n+1)} (K_u^{n+1} - K_1^{n+1}) \\ & + m_2 \times \frac{100}{\gamma} \frac{\sqrt{\gamma + \eta} \sqrt{\gamma + n}}{\sqrt{\gamma + \eta + n}} \end{aligned} \quad (5.18)$$

- (2) Mixed feeds of two beta distributions $f_1 (K; \gamma_1 = 2, \eta_1 = 3)$ and $f_2 (K; \gamma_2 = 3, \eta_2 = 2)$.

Its moments are:

$$\begin{aligned} \Theta_n = & m_1 \times 100 \frac{\sqrt{\gamma_1 + \eta_1} \sqrt{\gamma_1 + n}}{\sqrt{\gamma_1} \sqrt{\gamma_1 + \eta_1 + n}} + \\ & m_2 \times 100 \frac{\sqrt{\gamma_2 + \eta_2} \sqrt{\gamma_2 + n}}{\sqrt{\gamma_2} \sqrt{\gamma_2 + \eta_2 + n}} \end{aligned} \quad (5.19)$$

5.1.2. ANALYTICAL COMPUTATION OF HOLDUP :

The model equations for continuous cell and bank of cells are solved by Pearson's closure technique. The validity of technique is confirmed by comparing the Pearson's holdup with analytically calculated holdup. Exact holdup for a single flotation cell is computed as follows.

(1) Feed: Uniform Distribution

The exact total holdup at steady state is

$$\mu_o = \int_{K_1}^{K_u} M_p(K) dK = (h+1) \int_{K_1}^{K_u} \frac{\dot{F}(K)}{(hT + lT + K1)} dK \quad (5.20)$$

or

$$\mu_o = \frac{100(h+1)}{1(K_u - K_1)} \times \ln \left(\frac{hT + lT + K_u 1}{hT + lT + K_1 1} \right) \quad (5.21)$$

(2) Feed: Gamma Distribution

Exact holdup cannot be found out for gamma distribution. Therefore numerical integration was employed to estimate the value of holdup using trapezoidal rule algorithm.

(3) Feed: Beta Distribution

The exact total holdup at steady state is

$$\mu_o = \int_0^1 m_p(K) dK = (h+1) \int_0^1 \frac{\dot{F}(K)}{(hT + lT + Kl)} dK \quad (5.22)$$

$$\mu_o = 100 \frac{(h+1)}{\sqrt{\gamma} \sqrt{\eta}} \int_0^1 \frac{K^{\gamma-1} (1-K)^{\eta-1}}{(hT + lT + Kl)} dK \quad (5.23)$$

For $\gamma = 2$, $\eta = 3$ the expression is

$$\mu_o = 1200(h+1) \int_0^1 \frac{K(1-K)^2}{(hT + lT + Kl)} dK \quad (5.24)$$

For $\gamma = 3$, $\eta = 2$ the expression is

$$\mu_o = 1200(h+1) \int_0^1 \frac{K^2(1-K)}{(hT + lT + Kl)} dK \quad (5.25)$$

(4) Feed: Mixed Feeds Distribution

Exact holdup for mixed feeds of uniform and beta distribution is

$$\begin{aligned} \mu_o = & m_1 \times \frac{100(h+1)}{l(K_u - K_l)} \times \ln\left(\frac{hT + lT + K_u l}{hT + lT + K_l l}\right) \\ & + m_2 \times \frac{100(h+1)}{\sqrt{\gamma} \sqrt{\eta}} \int_0^1 \frac{K^{\gamma-1} (1-K)^{\eta-1}}{(hT + lT + Kl)} dK \end{aligned} \quad (5.26)$$

Exact holdup for mixed feeds of two beta distributions with different means is

$$\begin{aligned} \mu_o = & m_1 \times \frac{100(h+1)}{\sqrt{\gamma_1} \sqrt{\eta_1}} \int_0^1 \frac{K^{\gamma_1-1} (1-K)^{\eta_1-1}}{(hT + lT + Kl)} dK \\ & + m_2 \times \frac{100(h+1)}{\sqrt{\gamma_2} \sqrt{\eta_2}} \int_0^1 \frac{K^{\gamma_2-1} (1-K)^{\eta_2-1}}{(hT + lT + Kl)} dK \end{aligned} \quad (5.27)$$

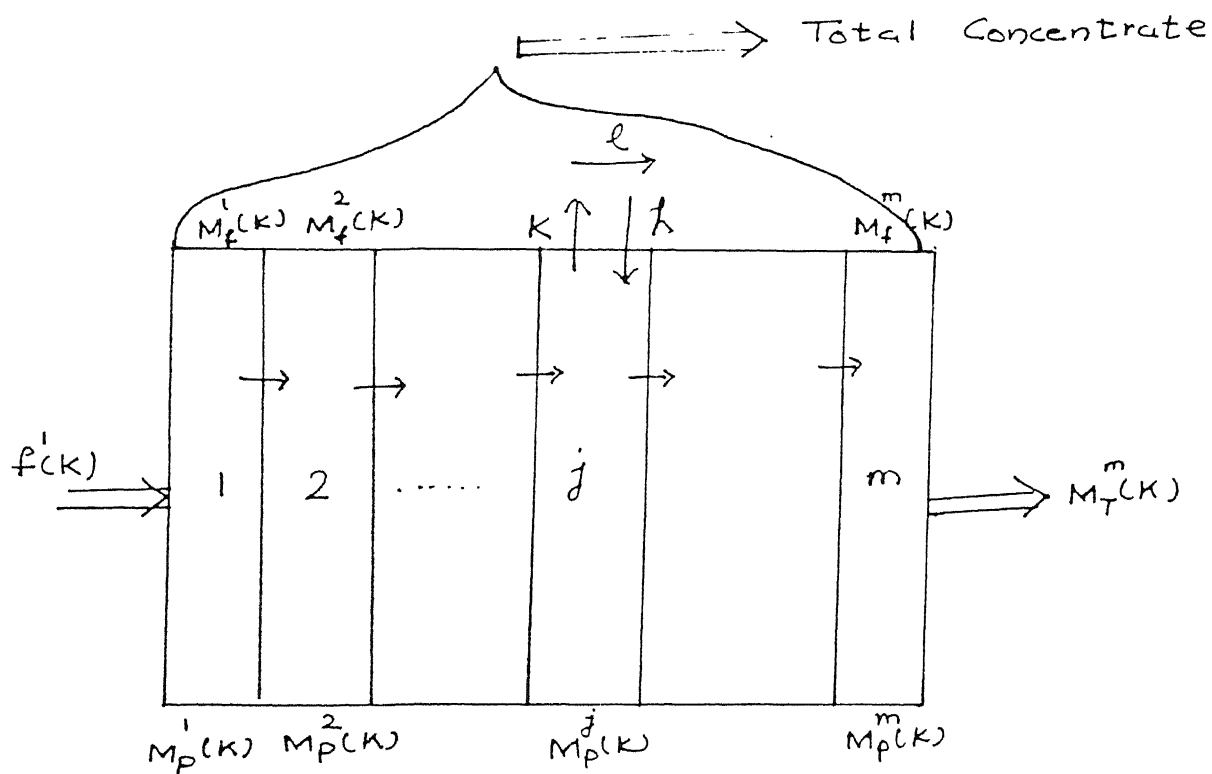


Fig 5.14 BANK OF FLOTATION CELLS.

Figures 5.1 to 5.6 show that the exact values of holdups are in good agreement with Pearson's holdups for the cases discussed above.

5.2. BANK OF FLOTATION CELLS :

Particulate feed to a flotation bank of cells has been characterised into 3 statistical distributions.

- (1) Uniform distribution
- (2) Beta distribution
- (3) Mixed feeds of two beta distributions.

Rates h , l and T are assumed to be same in each cell of the bank. ϕ_n is the feed to the first cell of the bank (Figure 5.14). Concentrate recovered from a bank is the sum of concentrates from each cell. Particulate feed flows from one cell to the next cell of the bank. An analytical expression is derived for computation of holdup in j th cell of the bank.

5.2.1. COMPUTATION OF EXACT HOLDUP :

In first cell of the bank, the model equations are (4.30) and (4.31).

$$\begin{aligned}
 M_p^1(K) &= \frac{(h + l)}{(hT + lT + Kl)} \dot{F}^1(K) \\
 M_T^1(K) &= T M_p^1(K) \\
 \dot{M}_T^1(K) &= \frac{(h + l)T}{(hT + lT + Kl)} \dot{F}^1(K)
 \end{aligned} \tag{5.28}$$

In second cell, the model equation is

$$M_p^2(K) = \frac{(h+1)}{(hT + lT + Kl)} M_T^1(K) \quad (5.29)$$

substitution of (5.28) yields

$$M_p^2(K) = \frac{(h+1)^2 T}{(hT + lT + Kl)^2} \dot{F}^1(K) \quad (5.30)$$

similarly for jth cell of the bank the expression is

$$M_p^j(K) = \frac{(h+1)^j T^{j-1}}{(hT + lT + Kl)^j} \dot{F}^1(K) \quad (5.31)$$

and total hold-up is

$$M_p^j = \int_0^\infty M_p^j(K) dK \quad (5.32)$$

The Pearson's and actual holdups are compared cell-wise in Figures 5.7 to 5.11. Pearson's holdups are in good agreement with exact value of holdups for different types of feeds to the bank.

The analytical expressions for total holdup (over all K's) in each cell are listed in appendix.

Thus validity of Pearson's closure technique for solving an incomplete set of moment equations has been demonstrated for a single cell and bank of cells.

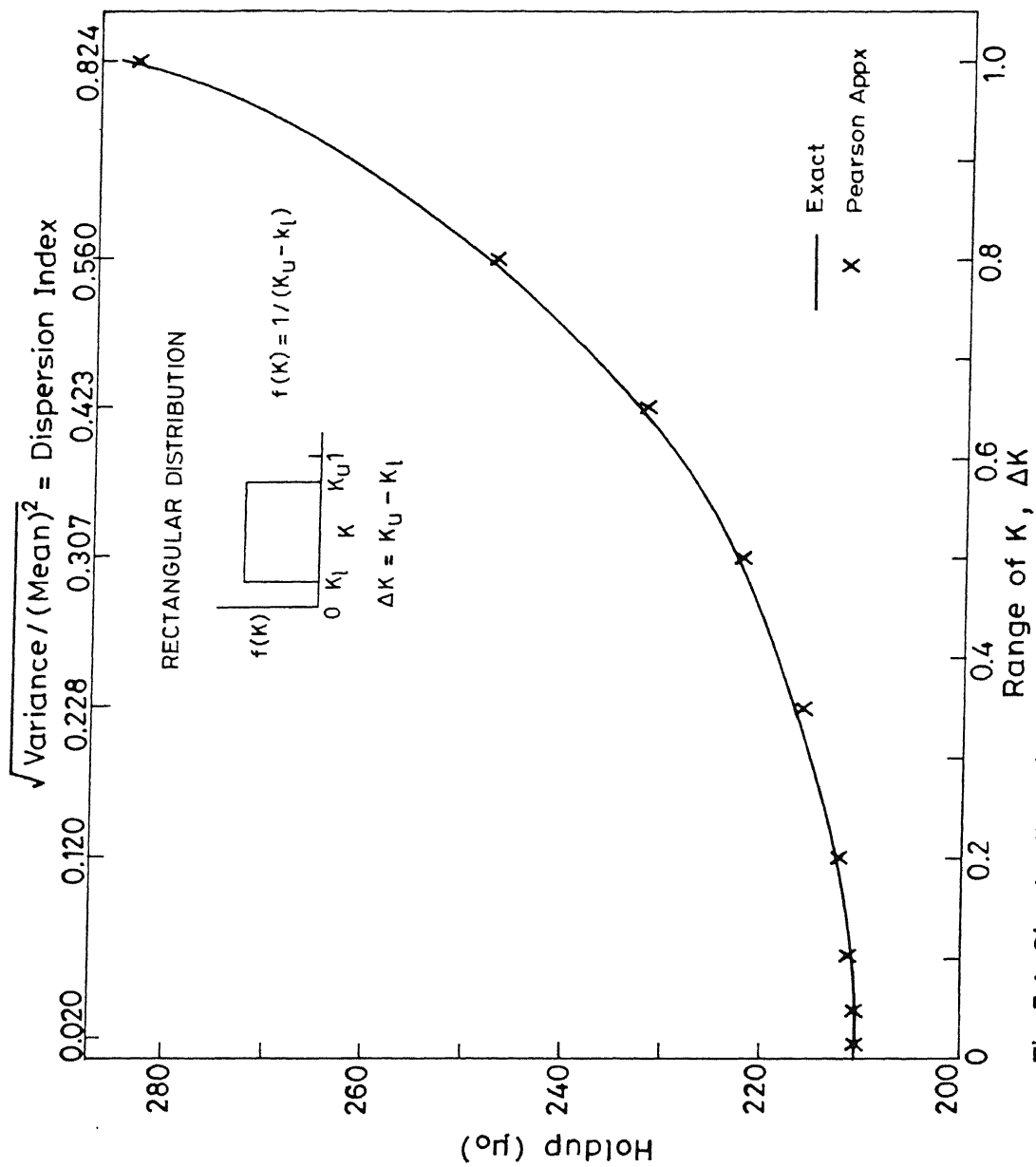


Fig.5.1 Single flotation cell-feed in rectangular distribution.

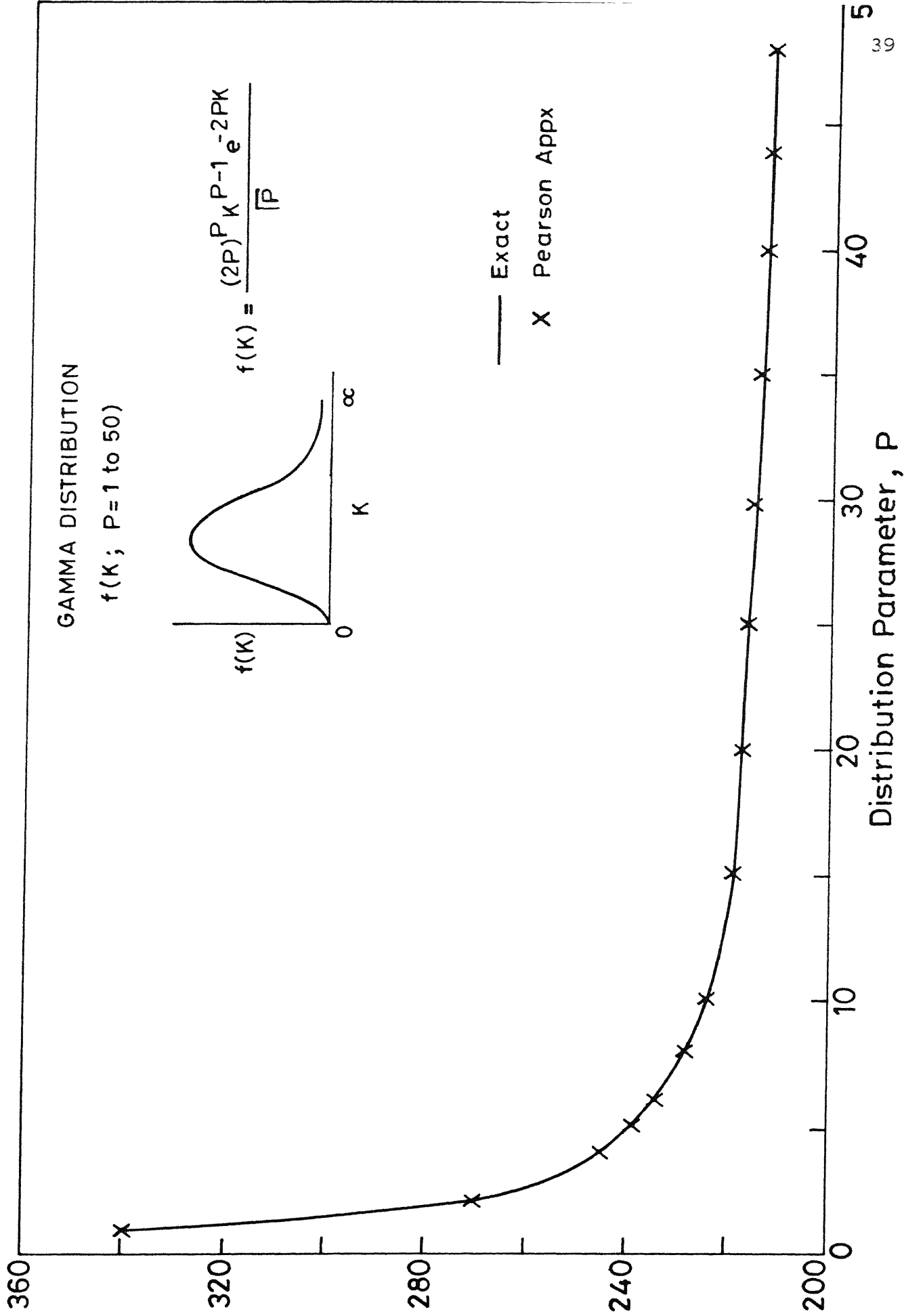


Fig.5.2 Single flotation cell-feed in gamma distribution

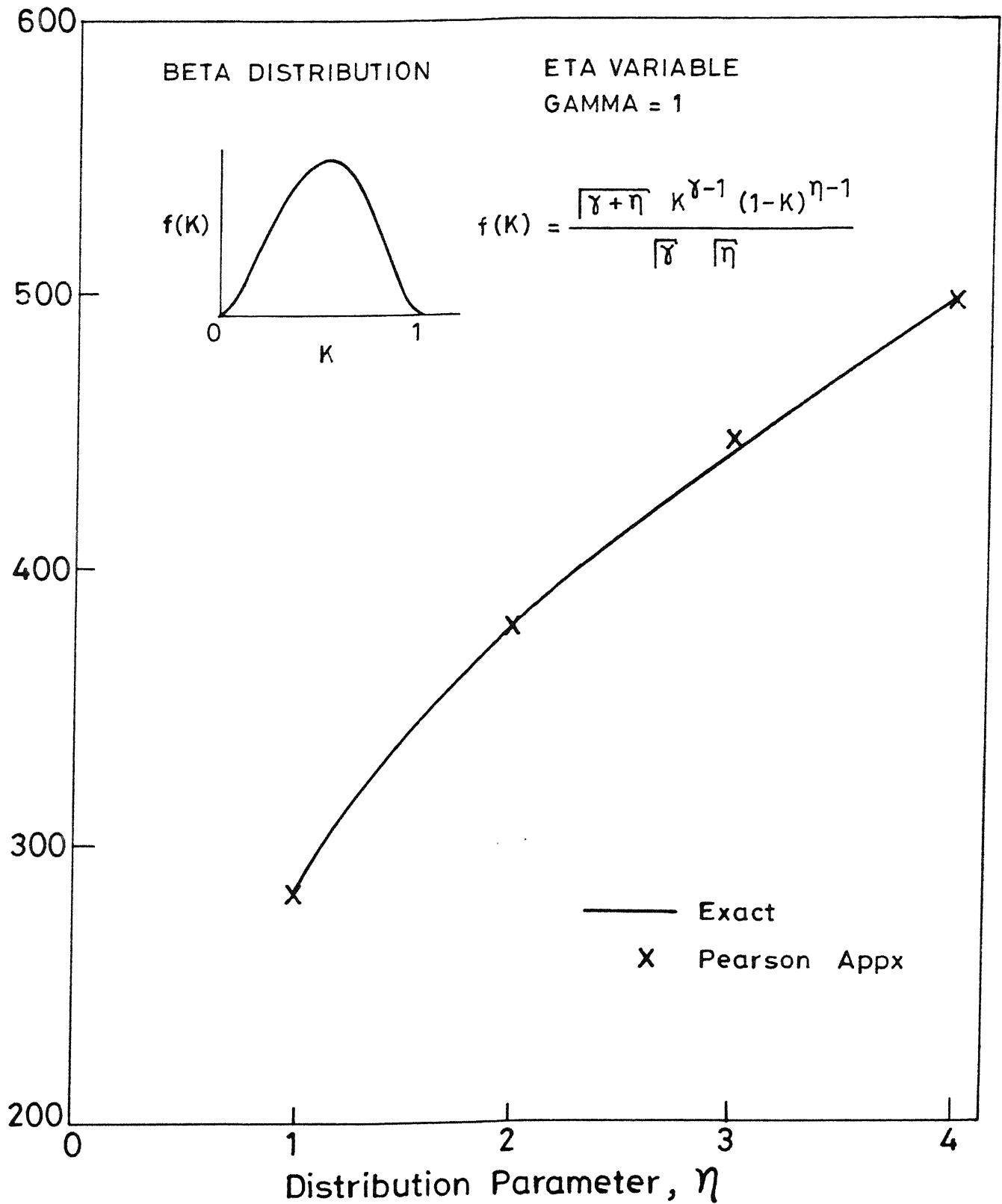


Fig.5.3 Single flotation cell-feed in beta distribution.

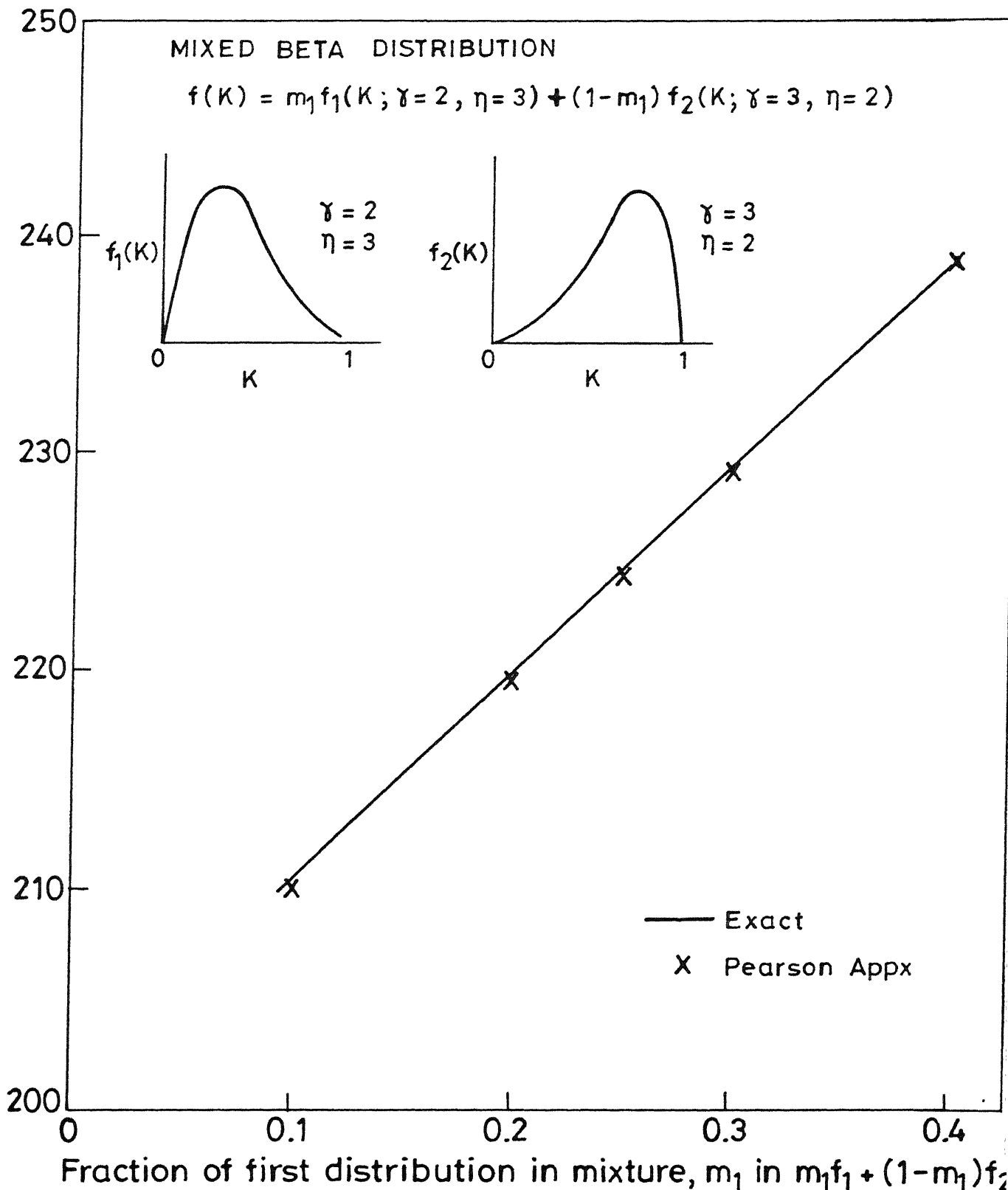


Fig.5.4 Single flotation cell-feed - two beta distributions mixed in different proportions.

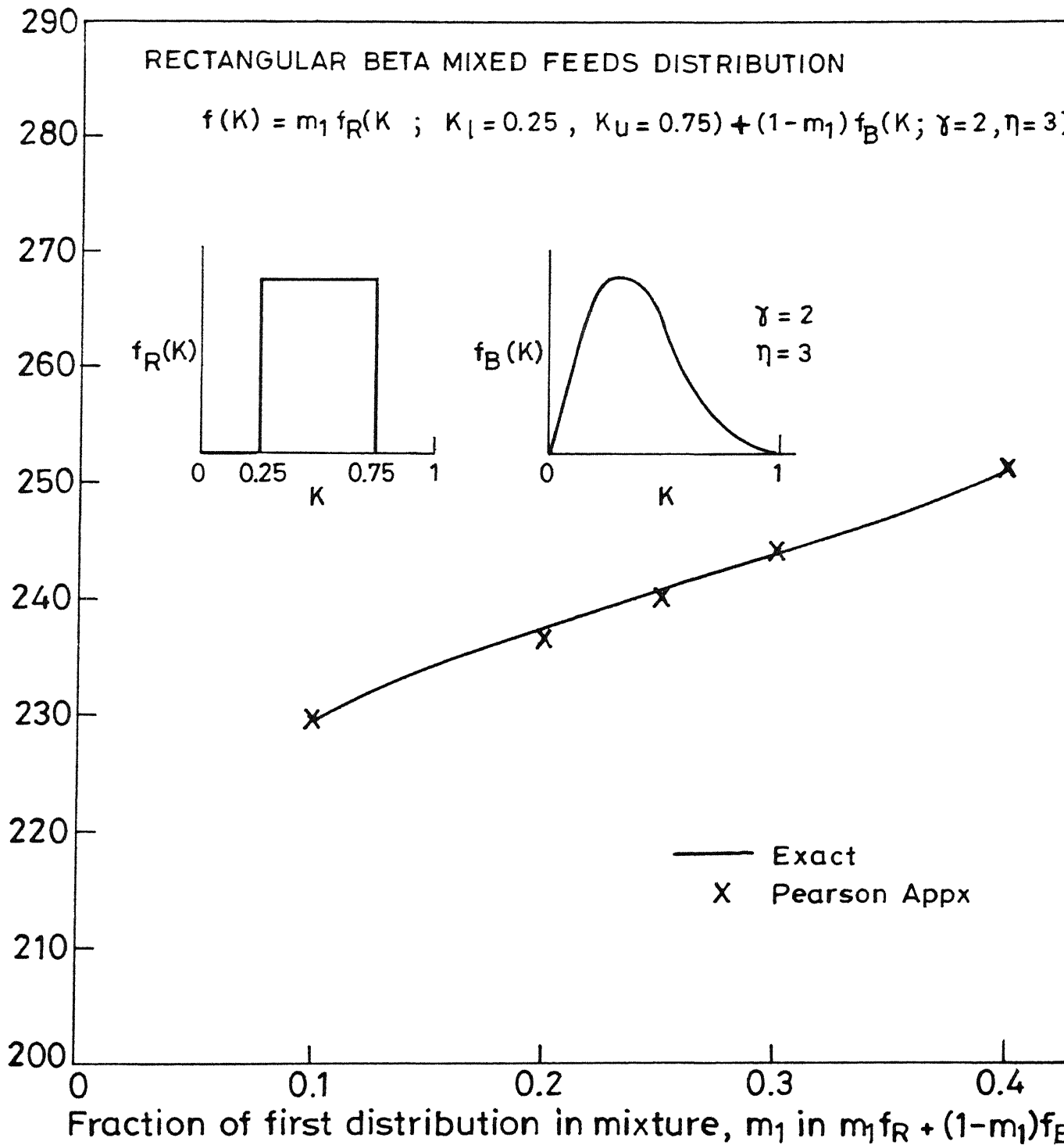
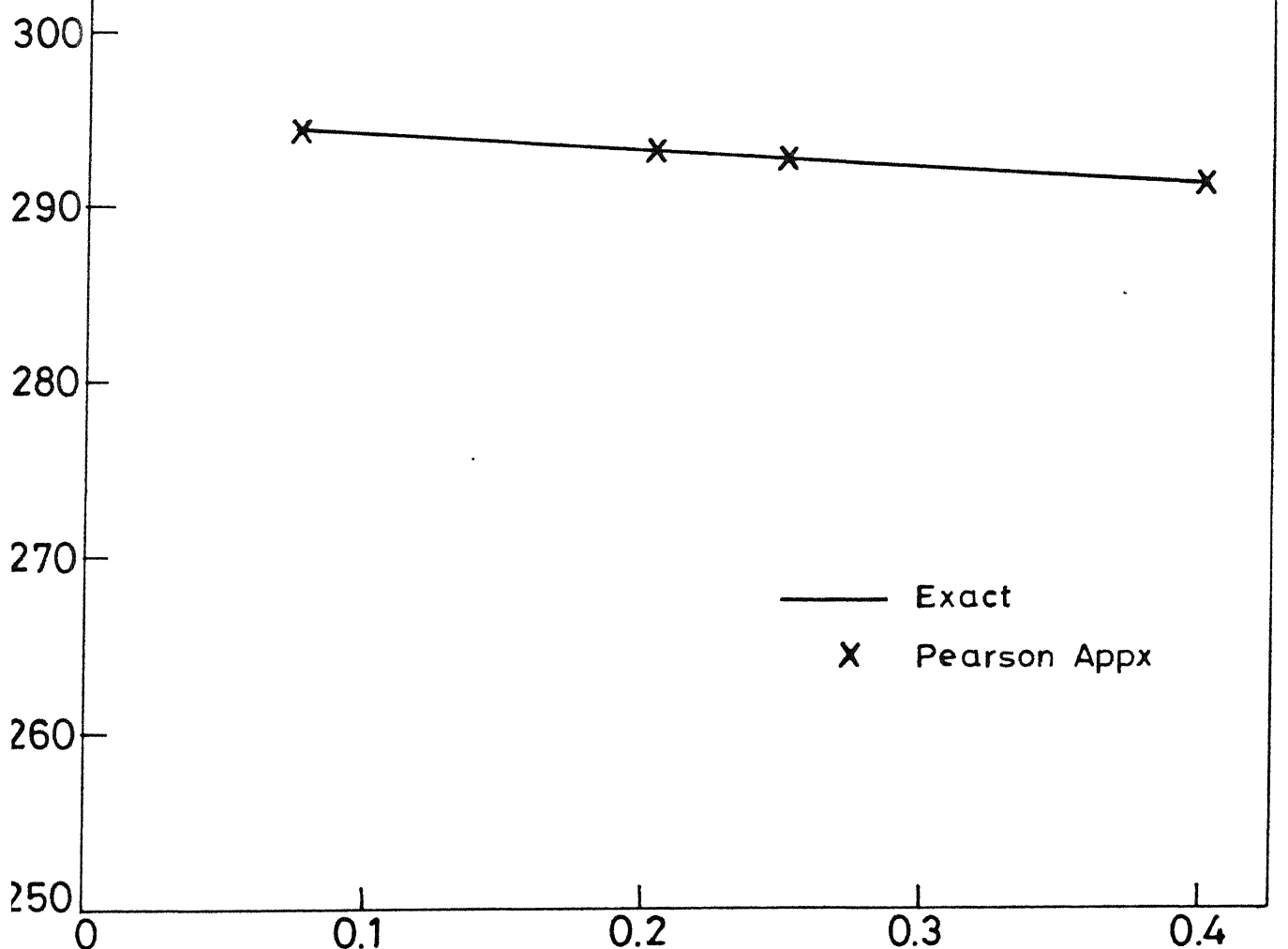
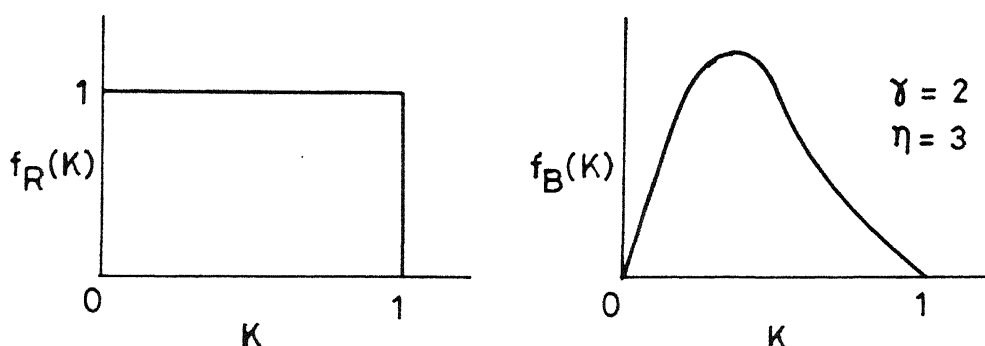


Fig.5.5 Single flotation cell-feed-beta and rectangular distributions mixed in different proportions.

RECTANGULAR AND BETA MIXED FEEDS DISTRIBUTION

$$f(K) = m_1 f_R(K; K_l=0, K_u=1) + (1-m) f_B(K; \gamma=2, \eta=3)$$



Fraction of first distribution in mixture, m_1 in $m_1 f_R + (1-m_1) f_B$

Fig.5.6 Single flotation cell-feed beta and rectangular distributions mixed in different proportions.

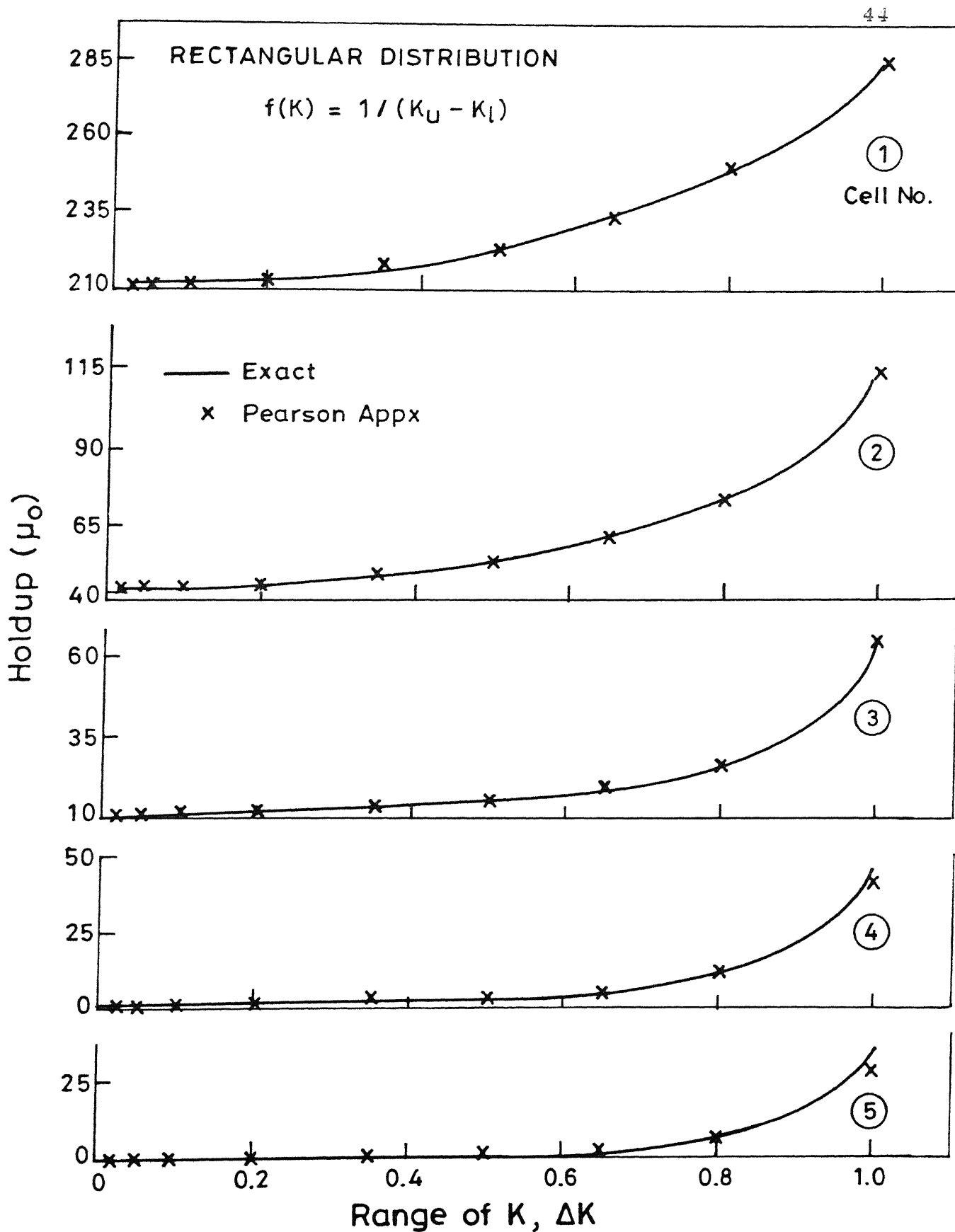


Fig.5.7 Single bank - rectangular distribution
 No of cells / bank = 5, $T = 0.1$

RECTANGULAR DISTRIBUTION

①
Cell No.

120

115

80

75

70

65

②

55

50

45

40

35

— Exact

x Pearson Appx

③

40

35

30

25

20

④

30

25

20

15

10

⑤

Range of K , ΔK Holdup (μ_0)

Figure 1. Holdup vs. range of K , ΔK for rectangular distribution

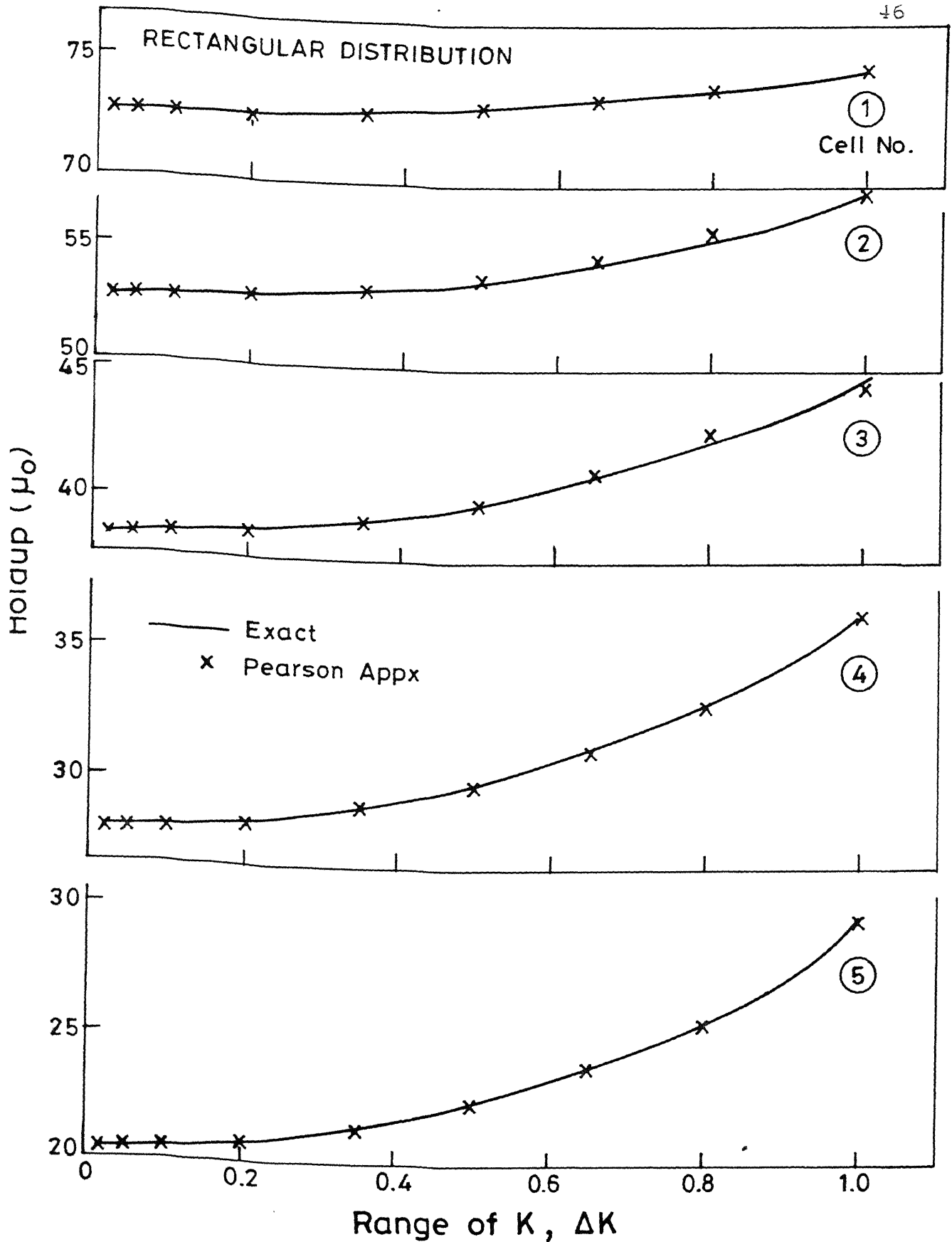


Fig.5.9 Single bank - rectangular distribution.
No of cells / bank = 5, $T = 1$

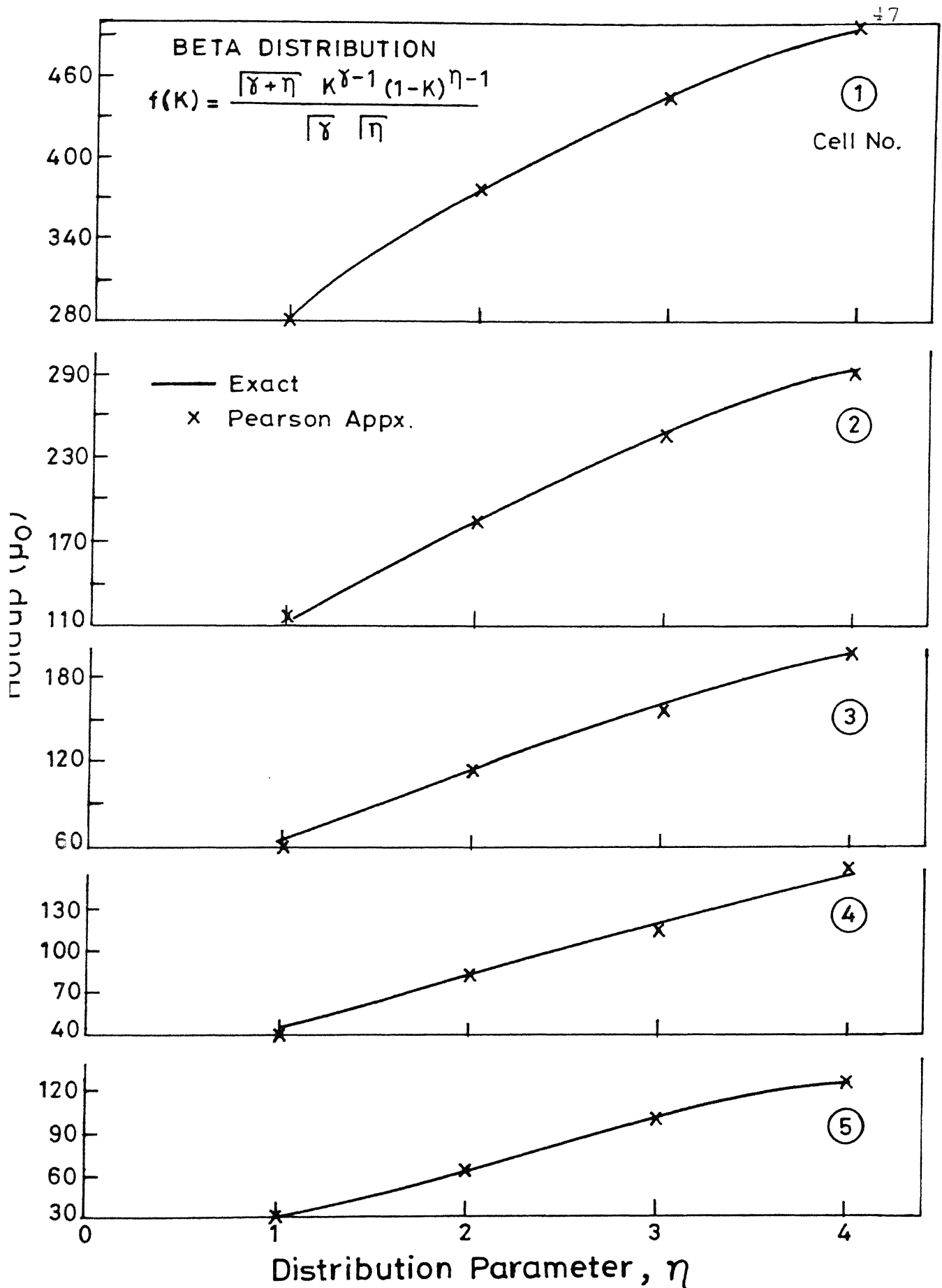
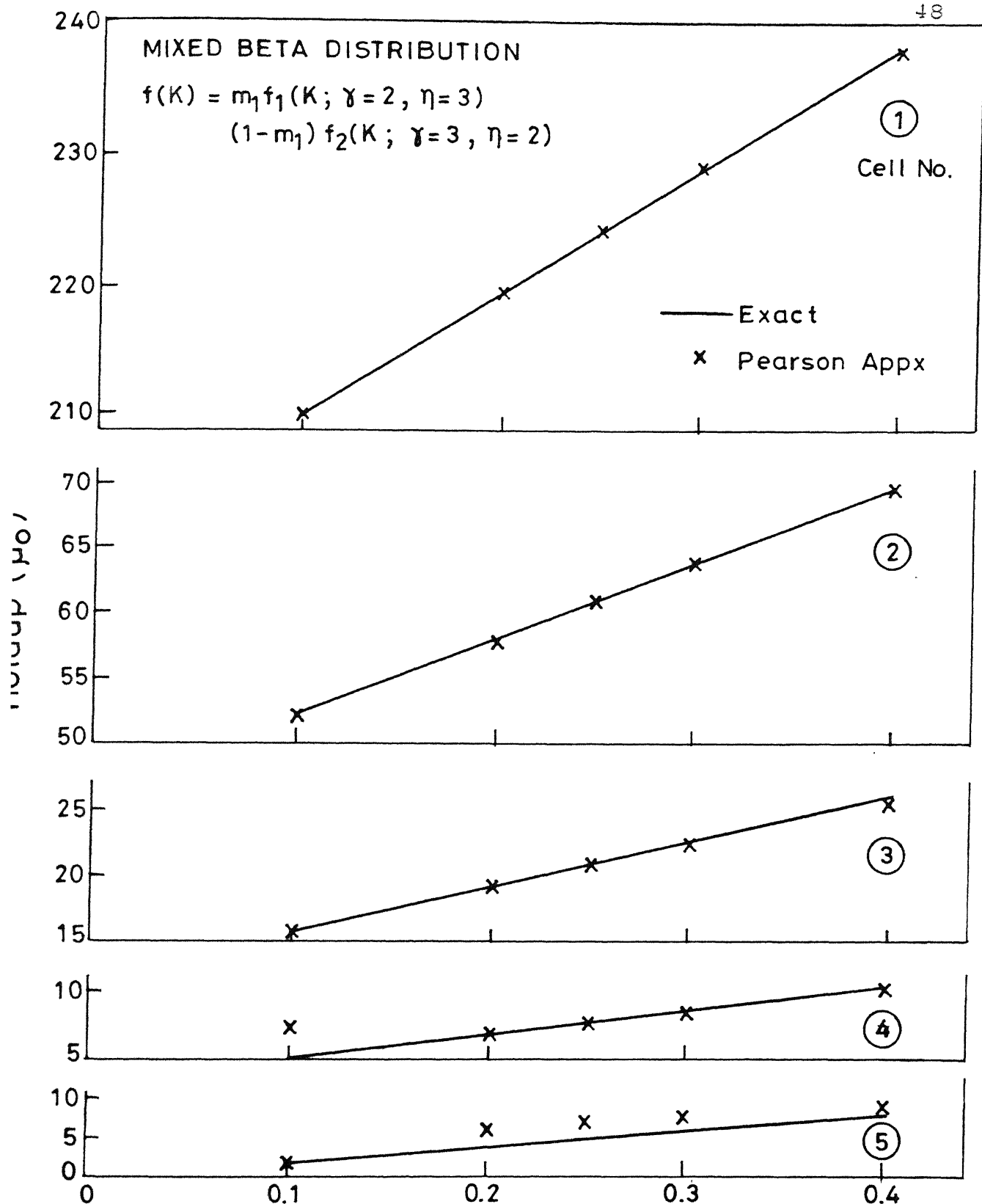


Fig. 5.10 Single bank - beta distribution (Gamma=1)



Fraction of first distribution in mixture, m_1 in $m_1 f_1 + (1-m_1) f_2$

Fig.5.11 Single bank - mixed beta distribution

No of cells/bank = 5, $T = 0.1$

BETA1 $\gamma=2, \eta=3$ BETA2 $\gamma=3, \eta=2$

CHAPTER 6

CONCLUSIONS

The Pearson's closure technique has been verified, using those functions for which analytical results are available in the previous chapter. Therefore we infer, the Pearson's closure technique for solving the incomplete set of moment equations, that arise in formulation of a flotation model, is quite accurate.

It is concluded that this could be a powerful and versatile method for modelling flotation circuits.

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DATE 31/10/1990

FOR CONTINUOUS TYPE OF FLOTATION CELL

h =1, l =3, T =0.1

Analytical expression for holdup in a single cell is

$$M_P(k) = \frac{(h+1)}{K_L(hT + lT + Kl)} F(k)$$

$$M_P = \int_{K_L}^{K_U} M_P(k) dK \quad \text{For feed in rectangular distribution}$$

$$M_P = \int_0^1 M_P(k) dK \quad \text{For feed in beta distribution}$$

FEED IN RECTANGULAR DISTRIBUTION
MEAN K=0.5

NO	INTERVAL	Ku -Kl	PEARSON HOLDUP	ERROR	EXACT HOLDUP
1	0.000-1.000	1.00	283.81	-0.0001	285.34
2	0.100-0.900	0.80	247.83	0.0001	248.01
3	0.175-0.825	0.65	232.58	0.0000	232.62
4	0.250-0.750	0.50	222.61	-0.0005	222.61
5	0.325-0.675	0.35	217.24	0.0001	216.14
6	0.400-0.600	0.20	212.42	0.0001	212.30
7	0.450-0.550	0.10	210.98	0.0001	210.97
8	0.475-0.525	0.05	210.64	0.0000	210.64
9	0.490-0.510	0.02	210.55	0.0001	210.54

FOR CONTINUOUS TYPE OF FLOTATION CELL

h =1, l =3, T =0.1

FEED IN GAMMA DISTRIBUTION
MEAN K=0.5

NO	N	PEARSON HOLDUP	ERROR	EXACT HOLDUP (NUMERICAL)
1	1	339.57	0.0000	339.55
2	2	279.93	0.0000	280.10
3	3	256.84	0.0000	256.90
4	4	245.03	0.0000	244.60
5	5	237.96	0.0000	236.14
6	6	233.26	0.0000	233.00
7	8	227.45	0.0000	227.72
8	10	223.99	0.0000	224.36
9	15	219.43	0.0000	220.06
10	20	217.18	0.0000	216.91
11	25	215.83	0.0000	215.49
12	30	214.94	0.0000	214.91
13	35	214.31	0.0000	214.84

FOR CONTINUOUS TYPE OF FLOTATION CELL

h =1, l =3, T =0.1

FEED IN BETA DISTRIBUTION

NO	GAMMA	ETA	MEAN K	PEARSON HOLDUP	ERROR	EXACT HOLDUP
1	1	1	0.50	283.8111	0.0000	285.4384
2	1	2	0.33	378.7507	0.0000	380.3270
3	1	3	0.25	445.1530	0.0000	446.5559
4	1	4	0.20	495.6951	0.0000	497.0039
5	2	1	0.67	190.4413	0.0000	190.5498
6	3	1	0.75	161.8711	-0.0003	161.8901
7	4	1	0.80	148.9964	-0.0001	149.0112
8	2	3	0.40	295.1752	0.0000	295.2119
9	3	2	0.60	200.5183	-0.0001	200.5265

FOR CONTINUOUS TYPE OF FLOTATION CELL

$h = 1, l = 3, T = 0.1$

MIXED FEED OF TWO BETA DISTRIBUTIONS

BETA1 - GAMMA=2 , ETA=3 , MEAN K= 0.4

BETA2 - GAMMA=3 , ETA=2 , MEAN K=0.6

BETA1 : BETA2 = $m_1:m_2$

NO	m_1	m_2	PEARSON HOLDUP	ERROR	EXACT HOLDUP
1	0.10	0.90	210.03	0.0000	210.00
2	0.20	0.80	219.52	0.0000	219.46
3	0.25	0.75	224.25	0.0000	224.20
4	0.30	0.70	228.97	0.0000	228.93
5	0.40	0.60	238.37	0.0001	238.40

FOR CONTINUOUS TYPE OF FLOTATION CELL

$h = 1, l = 3, T = 0.1$

MIXED FEED OF BETA AND RECTANGULAR DISTRIBUTION

BETA1 - GAMMA=2 , ETA=3 , MEAN K= 0.4

$K_1 = 0, K_u = 1$

RECTANGULAR : BETA1 = $m_1:m_2$

NO	m_1	m_2	PEARSON HOLDUP	ERROR	EXACT HOLDUP
1	0.10	0.90	294.16	0.0000	294.19
2	0.20	0.80	292.83	0.0000	293.21
3	0.25	0.75	292.13	0.0000	292.72
4	0.30	0.70	291.45	0.0000	292.22
5	0.40	0.60	290.11	0.0001	291.24

FOR CONTINUOUS TYPE OF FLOTATION CELL

$h = 1, l = 3, T = 0.1$

MIXED FEED OF BETA AND RECTANGULAR DISTRIBUTION

BETA1 - GAMMA=2 , ETA=3 , MEAN K= 0.4

K1= 0.25 , Ku=0.75

BETA1 : RECTANGULAR = m1:m2

NO	m1	m2	PEARSON HOLDUP	ERROR	EXACT HOLDUP
1	0.10	0.90	229.48	0.0000	229.86
2	0.20	0.80	236.64	0.0000	237.12
3	0.25	0.75	240.30	0.0000	240.74
4	0.30	0.70	243.99	0.0000	244.37
5	0.40	0.60	251.42	0.0000	251.63

Analytical expression for holdup in 1st cell of the bank is

$$M_P(k) = \frac{(h+1)}{(hT + 1T + K1)} \dot{F}(k)$$

$$M_P = \int_{K_1}^{K_u} M_P(k) dK \quad \text{For feed in rectangular distribution}$$

$$M_P = \int_0^1 M_P(k) dK \quad \text{For feed in beta distribution}$$

Analytical expression for holdup in 2nd cell of the bank is

$$M_P(k) = \frac{(h+1)^2 T}{(hT + 1T + K1)^2} \dot{F}(k)$$

$$M_P = \int_{K_1}^{K_u} M_P(k) dK \quad \text{For feed in rectangular distribution}$$

$$M_P = \int_0^1 M_P(k) dK \quad \text{For feed in beta distribution}$$

Analytical expression for holdup in 3rd cell of the bank is

$$M_P(k) = \frac{(h+1)^3 T^2}{(hT + 1T + K1)^3} \dot{F}(k)$$

$$M_P = \int_{K_1}^{K_u} M_P(k) dK \quad \text{For feed in rectangular distribution}$$

$$M_P = \int_0^1 M_P(k) dK \quad \text{For feed in beta distribution}$$

Analytical expression for holdup in 4th cell of the bank is

$$M_P(k) = \frac{(h+1)^4 T^3}{(hT + 1T + K1)^4} \dot{F}(k)$$

$$M_P = \int_{K_1}^{K_u} M_P(k) dK \quad \text{For feed in rectangular distribution}$$

$$M_P = \int_0^1 M_P(k) dK \quad \text{For feed in beta distribution}$$

Analytical expression for holdup in 5th cell of the bank is

$$M_P(k) = \frac{(h+1)^5 T^4}{(hT + 1T + K1)^5} \dot{F}(k)$$

$$M_P = \int_{K_1}^{K_u} M_P(k) dK \quad \text{For feed in rectangular distribution}$$

$$M_p = \int_0^1 M_p(k) dk \quad \text{For feed in beta distribution}$$

DATE 1/11/90

SINGLE BANK WITH 5 CELLS

UNIFORM DISTRIBUTION $K1 = 0.0$ $Ku = 1.0$ $h=1$; $l=3$; $T=0.1$

NO	MOMENT INDEX	FEED(Z)	HOLDUP (MEW0)	CONCENTRATE(ETA)	ERROR	EXACT HOLDUP
1	0	100.0000	283.8110	71.6189	0.00021	285.3420
	1	50.0000	95.4919	40.4508		
	2	33.3333	53.9344	27.9399		
	3	25.0000	37.2532	21.2747		
	4	20.0000	28.3662	17.1634		
	5	16.6667	22.8845	14.3782		
2	0	28.3811	113.3230	17.0488	0.00045	117.6470
	1	9.5492	22.7317	7.2760		
	2	5.3934	9.7014	4.4233		
	3	3.7253	5.8977	3.1355		
	4	2.8366	4.1807	2.4186		
	5	2.2885	3.2247	1.9660		
3	0	11.3323	60.7440	5.2579	0.05843	65.7440
	1	2.2732	7.0105	1.5721		
	2	0.9701	2.0962	0.7605		
	3	0.5898	1.0140	0.4884		
	4	0.4181	0.6512	0.3530		
	5	0.3225	0.4706	0.2754		
4	0	6.0744	39.5200	2.1224	0.00000	44.3720
	1	0.7011	2.8299	0.4181		
	2	0.2096	0.5574	0.1539		
	3	0.1014	0.2052	0.0809		
	4	0.0651	0.1078	0.0543		
	5	0.0471	0.0724	0.0398		
5	0	3.9520	28.6740	1.0846	0.00000	33.3270
	1	0.2830	1.4461	0.1384		
	2	0.0557	0.1845	0.0373		
	3	0.0205	0.0497	0.0155		
	4	0.0108	0.0207	0.0087		
	5	0.0072	0.0116	0.0061		

the tails in the bank are 2.8674
the total concentrate is 97.13260
the total holdup is 526.07200

SINGLE BANK WITH 5 CELLS

UNIFORM DISTRIBUTION $K1 = 0.1$ $Ku = 0.9$ $h=1$; $l=3$; $T=0.1$

NO	MOMENT INDEX	FEED(Z)	HOLDUP (MEW0)	CONCENTRATE(ETA)	ERROR	EXACT HOLDUP
1	0	100.0000	247.8330	75.2167	0.00067	248.0130
	1	50.0000	100.2889	39.9711		

2	0	30.3333	53.2948	25.0039		
3	1	20.5000	33.3385	17.1662		
4	2	14.7620	22.8882	12.4732		
5	3	11.0717	16.6309	9.4086		
<hr/>						
2	0	24.7833	73.3560	17.4477	0.00021	73.7330
	1	10.0289	23.2636	7.7025		
	2	5.3295	10.2700	4.3025		
	3	3.3338	5.7366	2.7602		
	4	2.2888	3.6802	1.9208		
	5	1.6631	2.5611	1.4070		
<hr/>						
3	0	7.3356	25.3600	4.7996	0.00036	25.8230
	1	2.3264	6.3995	1.6864		
	2	1.0270	2.2486	0.8021		
	3	0.5737	1.0695	0.4667		
	4	0.3680	0.6223	0.3058		
	5	0.2561	0.4077	0.2153		
<hr/>						
4	0	2.5360	12.0090	1.3351	0.00000	10.2470
	1	0.6399	1.7801	0.4619		
	2	0.2249	0.6159	0.1633		
	3	0.1070	0.2177	0.0852		
	4	0.0622	0.1136	0.0509		
	5	0.0408	0.0678	0.0340		
<hr/>						
5	0	1.2009	5.9520	0.6057	0.00389	4.4310
	1	0.1780	0.8076	0.0973		
	2	0.0616	0.1297	0.0486		
	3	0.0218	0.0648	0.0153		
	4	0.0114	0.0204	0.0093		
	5	0.0068	0.0124	0.0055		

the tails in the bank are 0.5952
the total concentrate is 99.40480
the total holdup is 364.51000

SINGLE BANK WITH 5 CELLS

UNIFORM DISTRIBUTION $K1 = 0.175$ $Ku = 0.825$

$h=1$; $l=3$; $T=0.1$

NO	MOMENT INDEX	FEED(Z)	HOLDUP (MEW0)	CONCENTRATE(ETA)	ERROR	EXACT HOLDUP
<hr/>						
1	0	100.0000	232.5830	76.7417	0.00100	232.6180
	1	50.0000	102.3223	39.7678		
	2	28.5208	53.0237	23.2185		
	3	17.7813	30.9579	14.6855		
	4	11.7544	19.5806	9.7963		
	5	8.0839	13.0618	6.7777		
<hr/>						
2	0	23.2583	60.1000	17.2483	0.00016	60.1650
	1	10.2322	22.9977	7.9325		
	2	5.3024	10.5766	4.2447		
	3	3.0958	5.6596	2.5298		
	4	1.9581	3.3731	1.6207		
	5	1.3062	2.1610	1.0901		
<hr/>						
3	0	6.0100	18.7660	4.1334	0.00000	17.1940
	1	2.2998	5.5112	1.7487		
	2	1.0577	2.3315	0.8245		
	3	0.5660	1.0993	0.4560		
	4	0.3373	0.6080	0.2765		
	5	0.2161	0.3687	0.1792		

4	0	1.8766	6.2650	1.2501	0.00950	5.3450
	1	0.5511	1.6668	0.3844		
	2	0.2332	0.5126	0.1819		
	3	0.1099	0.2425	0.0857		
	4	0.0608	0.1142	0.0494		
	5	0.0369	0.0658	0.0303		

5	0	0.6265	2.2040	0.4061	0.00001	1.7740
	1	0.1667	0.5415	0.1125		
	2	0.0513	0.1500	0.0363		
	3	0.0243	0.0483	0.0194		
	4	0.0114	0.0259	0.0088		
	5	0.0066	0.0118	0.0054		

the tails in the bank are 0.2204
the total concentrate is 99.77960
the total holdup is 319.91800

SINGLE BANK WITH 5 CELLS

UNIFORM DISTRIBUTION $K_1 = 0.25$ $K_u = 0.75$

$h=1$; $l=3$; $T=0.1$

NO	MOMENT INDEX	FEED(Z)	HOLDUP (MEW0)	CONCENTRATE(ETA)	ERROR	EXACT HOLDUP
1	0	100.0000	222.6070	77.7393	0.00003	222.6130
	1	50.0000	103.6524	39.6348		
	2	27.0833	52.8463	21.7987		
	3	15.6250	29.0649	12.7185		
	4	9.4531	16.9580	7.7573		
	5	5.9245	10.3431	4.8902		
2	0	22.2607	53.8540	16.8753	0.00000	52.5020
	1	10.3652	22.5004	8.1152		
	2	5.2846	10.8203	4.2026		
	3	2.9065	5.6035	2.3461		
	4	1.6958	3.1282	1.3830		
	5	1.0343	1.8440	0.8499		
3	0	5.3854	13.8530	4.0001	0.01643	13.0930
	1	2.2500	5.3335	1.7167		
	2	1.0820	2.2889	0.8531		
	3	0.5603	1.1375	0.4466		
	4	0.3128	0.5955	0.2533		
	5	0.1844	0.3377	0.1506		
4	0	1.3853	3.6140	1.0239	0.00153	3.4350
	1	0.5333	1.3652	0.3968		
	2	0.2289	0.5291	0.1760		
	3	0.1138	0.2346	0.0903		
	4	0.0595	0.1204	0.0475		
	5	0.0338	0.0633	0.0274		
5	0	0.3614	0.8940	0.2720	0.00000	0.9410
	1	0.1365	0.3627	0.1003		
	2	0.0529	0.1337	0.0395		
	3	0.0235	0.0527	0.0182		
	4	0.0120	0.0243	0.0096		
	5	0.0063	0.0128	0.0051		

the tails in the bank are 0.0894
the total concentrate is 99.91060
the total holdup is 294.82200

SINGLE BANK WITH 5 CELLS

UNIFORM DISTRIBUTION $K1 = 0.325$ $Ku = 0.675$ $h=1$; $l=3$; $T=0.1$

NO	MOMENT INDEX	FEED(Z)	HOLDUP (MEW0)	CONCENTRATE(ETA)	ERROR	EXACT HOLDUP
1	0	100.0000	217.2380	78.2762	0.00001	216.1440
	1	50.0000	104.3683	39.5632		
	2	26.0208	52.7509	20.7457		
	3	14.0313	27.6610	11.2652		
	4	7.8000	15.0202	6.2980		
	5	4.4479	8.3973	3.6082		
2	0	21.7238	49.4010	16.7837	0.00989	47.9850
	1	10.4368	22.3783	8.1990		
	2	5.2751	10.9320	4.1819		
	3	2.7661	5.5759	2.2085		
	4	1.5020	2.9447	1.2076		
	5	0.8397	1.6101	0.6787		
3	0	4.9401	12.2050	3.7196	0.00514	10.9370
	1	2.2378	4.9595	1.7419		
	2	1.0932	2.3225	0.8609		
	3	0.5576	1.1479	0.4428		
	4	0.2945	0.5904	0.2354		
	5	0.1610	0.3139	0.1296		
4	0	1.2205	3.2240	0.8981	0.00285	2.5560
	1	0.4959	1.1975	0.3762		
	2	0.2323	0.5016	0.1821		
	3	0.1148	0.2428	0.0905		
	4	0.0590	0.1207	0.0470		
	5	0.0314	0.0626	0.0251		
5	0	0.3224	0.3000	0.2924	-0.00002	0.6120
	1	0.1197	0.3899	0.0808		
	2	0.0502	0.1077	0.0394		
	3	0.0243	0.0525	0.0190		
	4	0.0121	0.0254	0.0095		
	5	0.0063	0.0127	0.0050		

the tails in the bank are 0.0300
the total concentrate is 99.97000
the total holdup is 282.36800

SINGLE BANK WITH 5 CELLS

UNIFORM DISTRIBUTION $K1 = 0.4$ $Ku = 0.6$ $h=1$; $l=3$; $T=0.1$

NO	MOMENT INDEX	FEED(Z)	HOLDUP (MEW0)	CONCENTRATE(ETA)	ERROR	EXACT HOLDUP
1	0	100.0000	212.4110	78.7589	0.00000	212.3020
	1	50.0000	105.0119	39.4988		
	2	25.3333	52.6651	20.0668		
	3	13.0000	26.7558	10.3244		
	4	6.7520	13.7659	5.3754		
	5	3.5467	7.1672	2.8299		
2	0	21.2411	45.5770	16.6834	0.00027	45.4550

1	10.5012	22.2445	8.2767		
2	5.2665	11.0356	4.1629		
3	2.6756	5.5506	2.1205		
4	1.3766	2.8274	1.0939		
5	0.7167	1.4585	0.5709		
<hr/>					
3	0	4.5577	9.9000	3.5677	0.00010 9.8140
	1	2.2245	4.7569	1.7488	
	2	1.1036	2.3317	0.8704	
	3	0.5551	1.1605	0.4390	
	4	0.2827	0.5853	0.2242	
	5	0.1458	0.2989	0.1160	
<hr/>					
4	0	0.9900	2.1860	0.7714	0.00004 2.1370
	1	0.4757	1.0285	0.3728	
	2	0.2332	0.4971	0.1835	
	3	0.1161	0.2446	0.0916	
	4	0.0585	0.1221	0.0463	
	5	0.0299	0.0618	0.0237	
<hr/>					
5	0	0.2186	0.3000	0.1886	-0.00031 0.4690
	1	0.1029	0.2515	0.0777	
	2	0.0497	0.1036	0.0394	
	3	0.0245	0.0525	0.0192	
	4	0.0122	0.0256	0.0097	
	5	0.0062	0.0129	0.0049	

the tails in the bank are 0.0300
the total concentrate is 99.97000
the total holdup is 270.37400

SINGLE BANK WITH 5 CELLS

UNIFORM DISTRIBUTION $K_1 = 0.45$ $K_u = 0.55$

$h=1$; $l=3$; $T=0.1$

NO	MOMENT INDEX	FEED(Z)	HOLDUP (MEW0)	CONCENTRATE(ETA)	ERROR	EXACT HOLDUP
<hr/>						
1	0	100.0000	210.9720	78.9028	0.00000	210.9650
	1	50.0000	105.2037	39.4796		
	2	25.0833	52.6395	19.8194		
	3	12.6250	26.4258	9.9824		
	4	6.3751	13.3099	5.0441		
	5	3.2295	6.7255	2.5569		
<hr/>						
2	0	21.0972	44.6070	16.6365	0.00001	44.5990
	1	10.5204	22.1820	8.3022		
	2	5.2640	11.0696	4.1570		
	3	2.6426	5.5427	2.0883		
	4	1.3310	2.7844	1.0525		
	5	0.6726	1.4034	0.5322		
<hr/>						
3	0	4.4607	9.4540	3.5153	0.00001	9.4480
	1	2.2182	4.6871	1.7495		
	2	1.1070	2.3327	0.8737		
	3	0.5543	1.1649	0.4378		
	4	0.2784	0.5837	0.2201		
	5	0.1403	0.2934	0.1110		
<hr/>						
4	0	0.9454	2.1000	0.7354	-0.00054	2.0060
	1	0.4687	0.9805	0.3707		
	2	0.2333	0.4942	0.1838		
	3	0.1165	0.2451	0.0920		
	4	0.0584	0.1226	0.0461		

	5	0.0293	0.0615	0.0232		
5	0	0.2100	0.3000	0.1800	0.00004	0.4270
	1	0.0981	0.2400	0.0741		
	2	0.0494	0.0987	0.0395		
	3	0.0245	0.0527	0.0192		
	4	0.0123	0.0257	0.0097		
	5	0.0061	0.0129	0.0049		

the tails in the bank are 0.0300
the total concentrate is 99.97000
the total holdup is 267.43300

SINGLE BANK WITH 5 CELLS

UNIFORM DISTRIBUTION $K_1 = 0.475$ $K_u = 0.525$

$h=1$; $l=3$; $T=0.1$

NO	MOMENT INDEX	FEED(Z)	HOLDUP (MEW0)	CONCENTRATE(ETA)	ERROR	EXACT HOLDUP
1	0	100.0000	210.6360	78.9364	0.00000	210.6360
	1	50.0000	105.2485	39.4751		
	2	25.0208	52.6335	19.7575		
	3	12.5313	26.3433	9.8969		
	4	6.2813	13.1959	4.9617		
	5	3.1511	6.6156	2.4895		
2	0	21.0636	44.3910	16.6245	0.00000	44.3900
	1	10.5249	22.1660	8.3083		
	2	5.2634	11.0777	4.1556		
	3	2.6343	5.5408	2.0803		
	4	1.3196	2.7737	1.0422		
	5	0.6616	1.3896	0.5226		
3	0	4.4391	9.3600	3.5031	-0.00001	9.3600
	1	2.2166	4.6708	1.7495		
	2	1.1078	2.3327	0.8745		
	3	0.5541	1.1660	0.4375		
	4	0.2774	0.5833	0.2190		
	5	0.1390	0.2920	0.1098		
4	0	0.9360	2.0000	0.7360	-0.00013	1.9750
	1	0.4671	0.9813	0.3689		
	2	0.2333	0.4919	0.1841		
	3	0.1166	0.2454	0.0921		
	4	0.0583	0.1227	0.0461		
	5	0.0292	0.0614	0.0231		
5	0	0.2000	0.3000	0.1700	-0.00001	0.4170
	1	0.0981	0.2267	0.0755		
	2	0.0492	0.1006	0.0391		
	3	0.0245	0.0522	0.0193		
	4	0.0123	0.0258	0.0097		
	5	0.0061	0.0129	0.0048		

the tails in the bank are 0.0300
the total concentrate is 99.97000
the total holdup is 266.68700

SINGLE BANK WITH 5 CELLS

UNIFORM DISTRIBUTION $K_1 = 0.49$ $K_u = 0.51$

$h=1$; $l=3$; $T=0.1$

NO	MOMENT INDEX	FEED(Z)	HOLDUP (MEW)	CONCENTRATE(ETA)	ERROR	EXACT HOLDUP
1	0	100.0000	210.5440	78.9456	0.00000	210.5440
	1	50.0000	105.2608	39.4739		
	2	25.0033	52.6319	19.7401		
	3	12.5050	26.3202	9.8730		
	4	6.2550	13.1640	4.9366		
	5	3.1292	6.5848	2.4707		
2	0	21.0544	44.3320	16.6212	-0.00001	44.3320
	1	10.5261	22.1616	8.3099		
	2	5.2632	11.0799	4.1552		
	3	2.6320	5.5403	2.0780		
	4	1.3164	2.7707	1.0393		
	5	0.6585	1.3858	0.5199		
3	0	4.4332	9.3350	3.4997	0.00000	9.3350
	1	2.2162	4.6663	1.7495		
	2	1.1080	2.3327	0.8747		
	3	0.5540	1.1663	0.4374		
	4	0.2771	0.5832	0.2187		
	5	0.1386	0.2917	0.1094		
4	0	0.9335	2.0000	0.7335	-0.00002	1.9660
	1	0.4666	0.9780	0.3688		
	2	0.2333	0.4918	0.1841		
	3	0.1166	0.2455	0.0921		
	4	0.0583	0.1228	0.0460		
	5	0.0292	0.0614	0.0230		
5	0	0.2000	0.3000	0.1700	0.00003	0.4140
	1	0.0978	0.2267	0.0751		
	2	0.0492	0.1002	0.0392		
	3	0.0245	0.0522	0.0193		
	4	0.0123	0.0258	0.0097		
	5	0.0061	0.0129	0.0048		

the tails in the bank are 0.0300
the total concentrate is 99.97000
the total holdup is 266.51100

DATE 1/11/90

SINGLE BANK WITH 5 CELLS

UNIFORM DISTRIBUTION $K1=0.0$ $Ku=1.0$ $h=1$; $l=3$; $T=0.5$

NO	MOMENT INDEX	FEED(Z)	HOLDUP (MEW0)	CONCENTRATE(ETA)	ERROR	EXACT HOLDUP
1	0	100.0000	122.1667	38.9166	-0.00012	122.1721
	1	50.0000	51.8889	24.0556		
	2	33.3333	32.0741	17.2963		
	3	25.0000	23.0617	13.4691		
	4	20.0000	17.9589	11.0206		
	5	16.6667	14.6941	9.3196		
2	0	61.0834	79.9752	21.0958	0.00027	80.0000
	1	25.9444	28.1277	11.8806		
	2	16.0370	15.8408	8.1167		
	3	11.5309	10.8222	6.1197		
	4	8.9794	8.1597	4.8996		
	5	7.3470	6.5328	4.0806		
3	0	39.9876	55.8189	12.0781	-0.51076	56.0000
	1	14.0638	16.1042	6.0118		
	2	7.9204	8.0157	3.9125		
	3	5.4111	5.2167	2.8027		
	4	4.0798	3.7370	2.2113		
	5	3.2664	2.9485	1.7922		
4	0	27.9095	41.0903	7.3643	0.00000	41.6000
	1	8.0521	9.8191	3.1425		
	2	4.0078	4.1901	1.9128		
	3	2.6004	2.5504	1.3332		
	4	1.8685	1.7775	0.9797		
	5	1.4742	1.3063	0.8211		
5	0	20.5451	31.5016	4.7943	0.00001	32.4800
	1	4.9095	6.3925	1.7133		
	2	2.0950	2.2844	0.9528		
	3	1.2752	1.2704	0.6400		
	4	0.8222	0.8533	0.4621		
	5	0.6532	0.6161	0.3451		

the tails in the bank are 15.7508

the total concentrate is 84.24920

the total holdup is 330.55269

SINGLE BANK WITH 5 CELLS

UNIFORM DISTRIBUTION $K1=0.1$ $Ku=0.9$ $h=1$; $l=3$; $T=0.5$

NO	MOMENT INDEX	FEED(Z)	HOLDUP (MEW0)	CONCENTRATE(ETA)	ERROR	EXACT HOLDUP
1	0	100.0000	119.1077	40.4462	0.00044	119.1089
	1	50.0000	53.9282	23.0359		
	2	30.3333	30.7145	14.9761		
	3	20.0000	19.9681	10.5160		
	4	14.7620	14.0213	7.7514		
	5	11.0717	10.3352	5.9041		

2	0	52.5539	74.0003	22.5537	0.00117	74.0056
	1	26.9641	30.0716	11.9283		
	2	15.3573	15.9044	7.4051		
	3	9.9410	9.8734	5.0473		
	4	7.9106	6.7798	3.6457		
	5	5.1676	4.8610	2.7371		

3	0	37.0001	47.9085	13.0459	0.09772	47.9222
	1	19.0358	17.3945	6.3386		
	2	7.9502	8.4514	3.7265		
	3	4.9367	4.9687	2.4524		
	4	3.3649	3.2698	1.7300		
	5	2.4395	2.3066	1.2772		

4	0	23.9543	32.1905	7.8590	-0.11782	32.2479
	1	8.072	10.4787	3.4579		
	2	4.2257	4.6105	1.9204		
	3	2.4843	2.5606	1.2040		
	4	1.6349	1.6054	0.8322		
	5	1.1533	1.1097	0.5985		

5	0	16.0952	22.3173	4.9366	0.00005	22.4568
	1	5.2394	6.5821	1.9483		
	2	2.3053	2.5977	1.0064		
	3	1.2903	1.3419	0.6094		
	4	0.8027	0.8125	0.3964		
	5	0.5548	0.5286	0.2905		

the tails in the bank are 11.1586
the total concentrate is 88.84136
the total holdup is 295.52422

SINGLE BANK WITH 5 CELLS

UNIFORM DISTRIBUTION $K1 = 0.175$ $Ku = 0.825$

$h=1$; $l=3$; $T=0.5$

NO	MOMENT INDEX	FEED(Z)	HOLDUP (MEW0)	CONCENTRATE(ETA)	ERROR	EXACT HOLDUP
1	0	100.0000	117.3874	41.3063	0.00081	117.3878
	1	50.0000	55.0750	22.4625		
	2	28.5208	29.9500	13.5458		
	3	17.7813	18.0611	8.7507		
	4	11.7544	11.6676	5.9206		
	5	8.0839	7.8941	4.1368		
2	0	58.6937	70.7990	23.2942	-0.00126	70.8004
	1	27.5375	31.0589	12.0081		
	2	14.9750	16.0107	6.9696		
	3	9.0306	9.2928	4.3842		
	4	5.8339	5.8455	2.9110		
	5	3.9471	3.8814	2.0064		
3	0	35.3095	44.0985	13.3503	-0.00017	43.8611
	1	19.5295	17.8004	6.6293		
	2	8.0054	8.8391	3.5858		
	3	4.6464	4.7811	2.2559		
	4	2.9228	3.0078	1.4189		
	5	1.9407	1.8918	0.9948		
4	0	22.0492	28.2916	7.9085	-0.18683	27.8749
	1	8.9002	10.5446	3.6279		
	2	4.4195	4.8372	2.0010		

3	2.3296	2.6679	1.0566		
4	1.5039	1.4088	0.7995		
5	0.9459	1.0660	0.4129		
5	0	14.1408	18.4557	4.9129	-0.00008 18.1395
1	5.2723	6.5506	1.9970		
2	2.4186	2.6627	1.0872		
3	1.3340	1.4496	0.6092		
4	0.7044	0.8122	0.2983		
5	0.5330	0.3977	0.3341		

the tails in the bank are 9.2279
the total concentrate is 90.77213
the total holdup is 277.02227

SINGLE BANK WITH 5 CELLS

UNIFORM DISTRIBUTION $K1= 0.25$ $Ku= 0.75$

$h=1$; $l=3$; $T=0.5$

NO	MOMENT INDEX	FEED(Z)	HOLDUP (MEW)	CONCENTRATE(ETA)	ERROR	EXACT HOLDUP
1	0	100.0000	116.2907	41.8546	0.00018	116.0848
	1	50.0000	55.8062	22.0969		
	2	27.0833	29.4625	12.3521		
	3	15.6250	16.4694	7.3903		
	4	9.4531	9.8537	4.5263		
	5	5.9245	6.0350	2.9070		
2	0	58.1454	68.7897	23.7505	-0.20031	68.4492
	1	27.9031	31.6674	12.0694		
	2	14.7313	16.0925	6.6850		
	3	8.2347	8.9133	3.7780		
	4	4.7269	5.0374	2.4082		
	5	3.0175	3.2109	1.4121		
3	0	34.3948	41.2803	13.7547	-0.00005	40.9963
	1	15.8337	18.3396	6.6639		
	2	8.0463	8.8852	3.6037		
	3	4.4567	4.8049	2.0542		
	4	2.5187	2.7390	1.1492		
	5	1.6054	1.5323	0.8393		
4	0	20.6401	24.9980	8.1411	-0.00014	24.9298
	1	9.1698	10.8548	3.7424		
	2	4.4426	4.9899	1.9477		
	3	2.4025	2.5969	1.1040		
	4	1.3695	1.4720	0.6335		
	5	0.7661	0.8446	0.3438		
5	0	12.4990	15.1610	4.9185	0.00009	15.3814
	1	5.4274	6.5580	2.1484		
	2	2.4949	2.8645	1.0627		
	3	1.2984	1.4169	0.5900		
	4	0.7360	0.7867	0.3427		
	5	0.4223	0.4569	0.1939		

the tails in the bank are 7.5805
the total concentrate is 92.41948
the total holdup is 266.51971

SINGLE BANK WITH 5 CELLS

UNIFORM DISTRIBUTION $K1= 0.325$ $Ku= 0.675$

$h=1$; $l=3$; $T=0.5$

NO	MOMENT INDEX	FEED(Z)	HOLDUP (HFWO)	CONCENTRATE(ETA)	ERROR	EXACT HOLDUP
1	0	100 0000	115.1546	42 3986	0.00016	115.1546
	1	50 0000	66.8093	21 7342		
	2	25 0000	33.4046	11 5313		
	3	14 0313	16.8766	6 3437		
	4	7 0000	8.4383	3 5709		
	5	4 4479	4.7692	2 0673		
2	0	57 6014	66.8093	24 1562	-0.04717	66.8093
	1	28 2658	33.4046	12 1616		
	2	14 4925	16.8766	6 3817		
	3	7 6876	8.4383	3 4331		
	4	4 2291	4.7692	1 9404		
	5	2 3806	2.3846	1 0870		
3	0	33 4452	39.0555	13 8908	0.00023	39.0555
	1	16 1041	19.5277	6 8502		
	2	8 1078	9.7639	3 5409		
	3	4 2645	4.8819	1 8739		
	4	2 2287	2.4409	1 0261		
	5	1.2936	1.1642	0 6095		
4	0	19 5643	23.0024	8 0468	-0.00026	23.0024
	1	9 2539	11.5012	4 0993		
	2	4 5668	5.7506	1 9739		
	3	2 3006	2.8753	1 0447		
	4	1 2626	1.3913	0 5661		
	5	0 6841	0.6940	0 3067		
5	0	11 5175	13.6470	4 7045	0.00016	13.6470
	1	5 3646	6.8235	2 2252		
	2	2 5929	3.4117	1 1074		
	3	1 3160	1.6765	0 5777		
	4	0 6964	0.6962	0 3113		
	5	0 3774	0.3773	0 1699		

the tails in the bank are 6.8130
the total concentrate is 93.1820
the total holdup is 257.88263

SINGLE BANK WITH 5 CELLS

UNIFORM DISTRIBUTION $T=0.4$ For 11

$h=1$; $l=3$; $T=0.5$

NO	MOMENT INDEX	FEED(Z)	HOLDUP (HFWO)	CONCENTRATE(ETA)	ERROR	EXACT HOLDUP
1	0	100 0000	114.5668	42 7141	-0.00005	114.5668
	1	50 0000	65.7895	21 5239		
	2	25 3333	32.8947	10 9241		
	3	12 0000	16.4474	5 6773		
	4	6 7500	8.2237	2 9671		
	5	3 5467	4.1118	1 5686		
2	0	57 2959	65.8014	24 3852	0.00378	65.7895
	1	29 4761	32.9007	12 2193		
	2	14 3493	16.4504	6 2031		
	3	7 3227	8.2252	3 1873		
	4	3 7849	4.1126	1 6600		

	5	1 9781	2.2133	0.8714		
3	0	32 2007	37 8904	13 9555	0.00008	37.8722
	1	16 2569	18 6073	6 9531		
	2	9 1462	9 2708	3.5108		
	3	4 1354	4.6811	1.7948		
	4	2 1249	2.3931	0.9283		
	5	1 1066	1.2378	0.4878		
4	0	18 2452	21 8774	8.0065	-0.00012	21.8548
	1	9 3037	10 6753	3.9660		
	2	4 6344	5 2880	1.9914		
	3	2.3405	2.6552	1.0129		
	4	1 1966	1.3506	0.5213		
	5	0.6189	0.6950	0.2714		
5	0	10 9397	12.6670	4.6052	0.00053	12.6424
	1	5 3377	6 1403	2.2675		
	2	2 6440	3.0234	1.1323		
	3	1.3276	1.5098	0.5727		
	4	0 6753	0 7636	0.2935		
	5	0.3475	0.3913	0.1518		

the tails in the bank are 6 3335
the total concentrate is 93 66650
the total holdup is 252.80788

SINGLE BANK WITH 5 CELLS

UNIFORM DISTRIBUTION $K_1 = 0.45$ $K_u = 0.55$

$h=1$; $l=3$; $T=0.5$

NO	MOMENT INDEX	FEED(Z)	HOLDUP (MEW0)	CONCENTRATE(ETA)	ERROR	EXACT HOLDUP
1	0	100.0000	114.3557	42.8221	-0.00015	114.3558
	1	50.0000	57.0962	21.4519		
	2	25.0833	28.6025	10.7821		
	3	12.6250	14.3761	5.4370		
	4	6.3751	7.2493	2.7505		
	5	3.2295	3.6673	1.3958		
2	0	57.1779	65.4261	24.4648	-0.00003	65.4263
	1	28.5481	32.6197	12.2382		
	2	14.3013	16.3177	6.1424		
	3	7.1800	8.1899	3.0931		
	4	3.6246	4.1241	1.5626		
	5	1.8337	2.0834	0.7919		
3	0	32.7131	37.4552	13.9855	0.00002	37.4552
	1	16.3099	18.6473	6.9862		
	2	8.1588	9.3149	3.5014		
	3	4.0950	4.6685	1.7607		
	4	2.0621	2.3476	0.8882		
	5	1.0417	1.1843	0.4496		
4	0	18.7276	21.4553	7.9999	-0.00017	21.4555
	1	9.3237	10.6666	3.9904		
	2	4.6575	5.3205	1.9972		
	3	2.3342	2.6629	1.0028		
	4	1.1738	1.3370	0.5053		
	5	0.5922	0.6737	0.2553		
5	0	10.7277	12.2970	4.5792	-0.00018	12.2979
	1	5.3333	6.1055	2.2805		

2	2.6603	3.0407	1.1399
3	1.3315	1.5199	0.5715
4	0.6685	0.7620	0.2875
5	0.3369	0.3833	0.1452

the tails in the bank are 6.1485
the total concentrate is 93.85150
the total holdup is 250.98934

SINGLE BANK WITH 5 CELLS

UNIFORM DISTRIBUTION $K_1 = 0.475$ $K_u = 0.525$ $h=1$; $l=3$; $T=0.5$

NO	MOMENT INDEX	FEED(Z)	HOLDUP (MEW0)	CONCENTRATE(ETA)	ERROR	EXACT HOLDUP
1	0	100.0000	114.3037	42.8481	0.00021	114.3032
	1	50.0000	57.1309	21.4346		
	2	25.0208	28.5794	10.7311		
	3	12.5313	14.3082	5.3772		
	4	6.2813	7.1696	2.6965		
	5	3.1511	3.5953	1.3534		
2	0	57.1519	65.3377	24.4830	0.00007	65.3361
	1	28.5654	32.6440	12.2434		
	2	14.2897	16.3246	6.1274		
	3	7.1541	8.1699	3.0691		
	4	3.5848	4.0922	1.5387		
	5	1.7977	2.0516	0.7719		
3	0	32.6688	37.3550	13.9914	0.00005	37.3521
	1	16.3220	18.6551	6.9944		
	2	8.1623	9.3259	3.4993		
	3	4.0850	4.6658	1.7521		
	4	2.0461	2.3361	0.8780		
	5	1.0258	1.1707	0.4404		
4	0	18.6775	21.3617	7.9967	0.00017	21.3571
	1	9.3276	10.6622	3.9965		
	2	4.6630	5.3286	1.9987		
	3	2.3329	2.6649	1.0004		
	4	1.1680	1.3339	0.5011		
	5	0.5854	0.6681	0.2513		
5	0	10.6808	12.2200	4.5708	0.00026	12.2134
	1	5.3311	6.0944	2.2839		
	2	2.6643	3.0452	1.1417		
	3	1.3324	1.5223	0.5713		
	4	0.6670	0.7617	0.2861		
	5	0.3341	0.3815	0.1433		

the tails in the bank are 6.1100
the total concentrate is 93.89000
the total holdup is 250.57808

SINGLE BANK WITH 5 CELLS

UNIFORM DISTRIBUTION $K_1 = 0.49$ $K_u = 0.51$ $h=1$; $l=3$; $T=0.5$

NO	MOMENT INDEX	FEED(Z)	HOLDUP (MEW0)	CONCENTRATE(ETA)	ERROR	EXACT HOLDUP
1	0	100.0000	114.2987	42.8556	0.00009	114.2885
	1	50.0000	57.1409	21.4296		
	2	25.0000	28.5720	10.7170		
	3	12.5050	14.2893	5.3604		
	4	6.2550	7.1472	2.6814		
	5	3.1292	3.5752	1.3416		
2	0	57.1444	65.3116	24.4986	0.00004	65.3109
	1	28.5704	32.6514	12.2447		
	2	14.2864	16.3263	6.1232		
	3	7.1416	8.1643	3.0625		
	4	3.5736	4.0833	1.5319		
	5	1.7876	2.0426	0.7663		
3	0	32.6558	37.3240	13.9938	-0.00016	37.3233
	1	16.3257	18.6584	6.9965		
	2	8.1631	9.3287	3.4988		
	3	4.0822	4.6650	1.7496		
	4	2.0416	2.3329	0.8752		
	5	1.0213	1.1670	0.4378		
4	0	18.6620	21.3299	7.9971	-0.00004	21.3297
	1	9.3292	10.6628	3.9978		
	2	4.6644	5.3304	1.9992		
	3	2.3325	2.6656	0.9997		
	4	1.1664	1.3330	0.4999		
	5	0.5835	0.6666	0.2502		
5	0	10.6649	12.1880	4.5709	-0.00014	12.1899
	1	5.3314	6.0945	2.2941		
	2	2.6652	3.0455	1.1424		
	3	1.3328	1.5232	0.5712		
	4	0.6665	0.7616	0.2857		
	5	0.3333	0.3809	0.1428		

the tails in the bank are 6.0940
the total concentrate is 93.90600
the total holdup is 250.44207

DATE 1/11/90

SINGLE BANK WITH 5 CELLS

UNIFORM DISTRIBUTION K1= 0.0 Ku= 1.0

h=1 ; l=3 ; T=1

NO	MOMENT INDEX	FEED(Z)	HOLDUP (MEW0)	CONCENTRATE(ETA)	ERROR	EXACT HOLDUP
1	0	100.0000	74.6153	25.3847	0.00310	74.6154
	1	50.0000	33.8463	16.1537		
	2	33.3333	21.5382	11.7951		
	3	25.0000	15.7268	9.2732		
	4	20.0000	12.3643	7.6357		
	5	16.6667	10.1810	6.4857		
2	0	74.6153	57.4291	17.1862	0.00001	57.1429
	1	33.8463	22.9149	10.9314		
	2	21.5382	14.5752	6.9630		
	3	15.7268	9.2840	6.4428		
	4	12.3643	8.5904	3.7739		
	5	10.1810	5.0319	5.1491		
3	0	57.4291	45.5014	11.9277	-3.02524	44.8980
	1	22.9149	15.9036	7.0113		
	2	14.5752	9.3484	5.2268		
	3	9.2840	6.9691	2.3149		
	4	8.5904	3.0866	5.5038		
	5	5.0319	7.3384	-2.3065		
4	0	45.5014	36.7281	8.7733	-0.00001	36.1516
	1	15.9036	11.6978	4.2058		
	2	9.3484	5.6077	3.7407		
	3	6.9691	4.9876	1.9814		
	4	3.0866	2.6419	0.4447		
	5	7.3384	0.5929	6.7455		
5	0	36.7281	29.6387	7.0894	-0.00001	29.7793
	1	11.6978	9.4526	2.2452		
	2	5.6077	2.9936	2.6141		
	3	4.9876	3.4854	1.5022		
	4	2.6419	2.0030	0.6389		
	5	0.5929	0.8519	-0.2590		

the tails in the bank are 29.6387
the total concentrate is 70.36133
the total holdup is 243.91254

SINGLE BANK WITH 5 CELLS

UNIFORM DISTRIBUTION K1= 0.1 Ku= 0.9

h=1 ; l=3 ; T=1

NO	MOMENT INDEX	FEED(Z)	HOLDUP (MEW0)	CONCENTRATE(ETA)	ERROR	EXACT HOLDUP
1	0	100.0000	74.0563	25.9437	0.00000	73.9154
	1	50.0000	34.5916	15.4084		
	2	30.3333	20.5445	9.7888		
	3	20.5000	13.0518	7.4482		

4		14.7620	9.9310	4.8310		
5		11.0717	6.4414	4.6303		
<hr/>						
2	0	74.0563	55.8323	18.2240	-1.67117	55.5363
	1	34.5916	24.2986	10.2930		
	2	20.5445	13.7240	6.8205		
	3	13.0519	9.0939	3.9578		
	4	9.9310	5.2771	4.6538		
	5	6.4414	6.2051	0.2363		
<hr/>						
3	0	55.8323	42.7213	13.1110	0.00002	42.4088
	1	24.2986	17.4814	6.8172		
	2	13.7240	9.0896	4.6344		
	3	9.0939	6.1792	2.9148		
	4	5.2771	3.8863	1.3908		
	5	6.2051	1.8544	4.3507		
<hr/>						
4	0	42.7213	32.9878	9.7335	0.00000	32.8982
	1	17.4814	12.9780	4.5033		
	2	9.0896	6.0045	3.0852		
	3	6.1792	4.1136	2.0656		
	4	3.8863	2.7542	1.1322		
	5	1.8544	1.5096	0.3448		
<hr/>						
5	0	32.9878	25.4949	7.4929	0.00000	25.9067
	1	12.9780	9.9905	2.9876		
	2	6.0045	3.9834	2.0210		
	3	4.1136	2.6947	1.4189		
	4	2.7542	1.8918	0.8623		
	5	1.5096	1.1498	0.3598		

the tails in the bank are 25.4949
the total concentrate is 74.50510
the total holdup is 231.09255

SINGLE BANK WITH 5 CELLS

UNIFORM DISTRIBUTION $K1 = 0.175$ $Ku = 0.825$

h=1 ; l=3 ; T=1

NO	MOMENT INDEX	FEEU(Z)	HOLDUP (MEW0)	CONCENTRATE(ETA)	ERROR	EXACT HOLDUP
1	0	100.0000	73.5638	26.4362	0.00001	73.5038
	1	50.0000	35.2483	14.7517		
	2	28.5208	19.6690	8.8519		
	3	17.7813	11.8025	5.9788		
	4	11.7544	7.9717	3.7827		
	5	8.0839	5.0436	3.0403		
<hr/>						
2	0	73.5638	54.7363	18.8275	-0.71777	54.6087
	1	35.2433	25.1033	10.1450		
	2	19.6690	13.5266	6.1424		
	3	11.8025	8.1898	3.6127		
	4	7.9717	4.8169	3.1548		
	5	5.0436	4.2064	0.8372		
<hr/>						
3	0	54.7363	41.1461	13.5902	-0.00001	41.0040
	1	25.1033	18.1203	6.9830		
	2	13.5266	9.3107	4.2159		
	3	8.1898	5.6212	2.5686		
	4	4.8169	3.4247	1.3921		
	5	4.2064	1.8562	2.3502		

4	0	41.1461	31.1786	9.9676	0.00001	31.1111
	1	12.1203	13.2901	4.8302		
	2	9.3107	6.4403	2.8704		
	3	5.6212	3.8272	1.7940		
	4	3.4247	2.3920	1.0327		
	5	1.8562	1.3770	0.4792		

5	0	31.1786	23.7398	7.4387	0.00000	23.8447
	1	13.2701	9.9183	3.3718		
	2	6.4403	4.4957	1.9446		
	3	3.8272	2.5928	1.2345		
	4	2.3920	1.6460	0.7461		
	5	1.3770	0.9947	0.3822		

the tails in the bank are 23.7398
the total concentrate is 76.26016
the total holdup is 224.36465

SINGLE BANK WITH 5 CELLS

UNIFORM DISTRIBUTION $K_1 = 0.25$ $K_u = 0.75$

$h=1$; $l=3$; $T=1$

NO	MOMENT INDEX	FEED(Z)	HOLDUP (MEW0)	CONCENTRATE(ETA)	ERROR	EXACT HOLDUP
1	0	100.0000	73.2038	26.7962	0.00000	73.1832
	1	50.0000	35.7283	14.2717		
	2	27.0833	19.0290	8.0544		
	3	15.6250	10.7392	4.8858		
	4	9.4531	6.5145	2.9387		
	5	5.9245	3.9182	2.0063		
2	0	73.2038	53.9390	19.2648	-0.24796	53.8947
	1	35.7283	25.6865	10.0418		
	2	19.0290	13.3891	5.6399		
	3	10.7392	7.5198	3.2193		
	4	6.5145	4.2924	2.2220		
	5	3.9182	2.9627	0.9555		
3	0	53.9390	39.9896	13.9493	0.00001	39.9388
	1	25.6865	18.5991	7.0873		
	2	13.3891	9.4498	3.9393		
	3	7.5198	5.2524	2.2674		
	4	4.2924	3.0232	1.2693		
	5	2.9627	1.6924	1.2703		
4	0	39.9896	29.8095	10.1801	-0.00001	29.7802
	1	18.5991	13.5735	5.0256		
	2	9.4498	6.7008	2.7490		
	3	5.2524	3.6653	1.5872		
	4	3.0232	2.1162	0.9070		
	5	1.6924	1.2093	0.4831		
5	0	29.8095	22.3189	7.4906	0.00086	22.3406
	1	13.5735	9.9874	3.5861		
	2	6.7008	4.7814	1.9194		
	3	3.6653	2.5592	1.1061		
	4	2.1162	1.4748	0.6414		
	5	1.2093	0.8553	0.3540		

the tails in the bank are 22.3189
the total concentrate is 77.68111
the total holdup is 219.26072

SINGLE BANK WITH 5 CELLS

UNIFORM DISTRIBUTION $K1= 0.325$ $Ku= 0.675$ $h=1$; $l=3$; $T=1$

NO	MOMENT INDEX	FEED(Z)	HOLDUP (MEU0)	CONCENTRATE(ETA)	ERROR	EXACT HOLDUP
1	0	100.0000	72.9543	27.0457	0.00003	72.9494
	1	50.0000	36.0610	13.9390		
	2	26.0208	18.5854	7.4355		
	3	14.0313	9.9139	4.1173		
	4	7.8000	5.4897	2.3103		
	5	4.4479	3.0804	1.3676		
2	0	72.9543	53.3943	19.5600	0.04255	53.3789
	1	36.0610	26.0800	9.9810		
	2	18.5854	13.3079	5.2774		
	3	9.9139	7.0366	2.8774		
	4	5.4897	3.8365	1.6533		
	5	3.0804	2.2043	0.8760		
3	0	53.3943	39.2075	14.1867	0.02204	39.1780
	1	26.0800	18.9156	7.1644		
	2	13.3079	9.5525	3.7554		
	3	7.0366	5.0072	2.0294		
	4	3.8365	2.7058	1.1306		
	5	2.2043	1.5075	0.6968		
4	0	39.2075	28.8899	10.3177	0.00000	28.8424
	1	18.9156	13.7569	5.1587		
	2	9.5525	6.8783	2.6743		
	3	5.0072	3.5657	1.4415		
	4	2.7058	1.9220	0.7838		
	5	1.5075	1.0451	0.4624		
5	0	28.8899	21.3623	7.5275	-0.02774	21.2974
	1	13.7569	10.0367	3.7202		
	2	6.8783	4.9603	1.9180		
	3	3.5657	2.5573	1.0084		
	4	1.9220	1.3445	0.5775		
	5	1.0451	0.7700	0.2751		

the tails in the bank are 21.3623
the total concentrate is 78.63767
the total holdup is 215.80827

SINGLE BANK WITH 5 CELLS

UNIFORM DISTRIBUTION $K1= 0.4$ $Ku= 0.6$ $h=1$; $l=3$; $T=1$

NO	MOMENT INDEX	FEED(Z)	HOLDUP (MEU0)	CONCENTRATE(ETA)	ERROR	EXACT HOLDUP
1	0	100.0000	72.8003	27.1997	0.00094	72.7995
	1	50.0000	36.2663	13.7337		
	2	25.3333	18.3116	7.0217		
	3	13.0000	9.3623	3.6377		
	4	6.7500	4.8502	1.9018		
	5	3.5467	2.5357	1.0110		

2	0	72.8003	53.0528	19.7475	0.00128	53.0504
	1	36.2663	26.3300	9.9363		
	2	18.3116	13.2484	5.0631		
	3	9.3623	6.7509	2.6115		
	4	4.8502	3.4819	1.3683		
	5	2.5357	1.8244	0.7113		
3	0	53.0528	38.7019	14.3509	0.00177	38.6972
	1	26.3300	19.1345	7.1955		
	2	13.2484	9.5940	3.6545		
	3	6.7509	4.8726	1.8782		
	4	3.4819	2.5043	0.9776		
	5	1.8244	1.3035	0.5209		
4	0	38.7019	28.2630	10.4389	0.00052	28.2554
	1	19.1345	13.9186	5.2159		
	2	9.5940	6.9545	2.6394		
	3	4.8726	3.5192	1.3534		
	4	2.5043	1.8045	0.6998		
	5	1.3035	0.9331	0.3704		
5	0	28.2630	20.6627	7.6003	0.00073	20.6516
	1	13.9186	10.1337	3.7849		
	2	6.9545	5.0465	1.9080		
	3	3.5192	2.5441	0.9752		
	4	1.8045	1.3002	0.5043		
	5	0.9331	0.6724	0.2607		

the tails in the bank are 20.6627
the total concentrate is 79.33728
the total holdup is 213.48073

SINGLE BANK WITH 5 CELLS

UNIFORM DISTRIBUTION $K_1 = 0.45$ $K_u = 0.55$

$h=1$; $l=3$; $T=1$

NO	MOMENT INDEX	FEED(Z)	HOLDUP (MEW0)	CONCENTRATE(ETA)	ERROR	EXACT HOLDUP
1	0	100.0000	72.7453	27.2547	0.00001	72.7453
	1	50.0000	36.3395	13.6605		
	2	25.0833	18.2140	6.8694		
	3	12.6250	9.1592	3.4658		
	4	6.3751	4.6211	1.7540		
	5	3.2295	2.3387	0.8908		
2	0	72.7453	52.9319	19.8135	-0.00095	52.9319
	1	36.3395	26.4180	9.9215		
	2	18.2140	13.2287	4.9852		
	3	9.1592	6.6470	2.5122		
	4	4.6211	3.3496	1.2715		
	5	2.3387	1.6953	0.6434		
3	0	52.9319	38.5241	14.4077	0.00000	38.5246
	1	26.4180	19.2103	7.2077		
	2	13.2287	9.6103	3.6185		
	3	6.6470	4.8246	1.8223		
	4	3.3496	2.4298	0.9199		
	5	1.6953	1.2265	0.4688		
4	0	38.5241	28.0445	10.4796	0.00000	28.0457
	1	19.2103	13.9728	5.2375		

2	9.6103	6.9833	2.6269
3	4.8246	3.5026	1.3220
4	2.4298	1.7627	0.6671
5	1.2265	0.8894	0.3370

5	0	28.0445	20.4200	7.6245	0.00000	20.4222
	1	13.9728	10.1660	3.8068		
	2	6.9833	5.0758	1.9075		
	3	3.5026	2.5434	0.9592		
	4	1.7627	1.2790	0.4838		
	5	0.8894	0.6450	0.2444		

the tails in the bank are 20.4200
the total concentrate is 79.57996
the total holdup is 212.66590

SINGLE BANK WITH 5 CELLS

UNIFORM DISTRIBUTION $K_1 = 0.475$ $K_u = 0.525$

$h=1$; $l=3$; $T=1$

NO	MOMENT INDEX	FEED(Z)	HOLDUP (MEW0)	CONCENTRATE(ETA)	ERROR	EXACT HOLDUP
1	0	100.0000	72.7318	27.2682	0.00001	72.7318
	1	50.0000	36.3576	13.6424		
	2	25.0208	18.1898	6.8310		
	3	12.5313	9.1080	3.4233		
	4	6.2813	4.5643	1.7169		
	5	3.1511	2.2892	0.8618		
2	0	72.7318	52.9024	19.8294	0.00001	52.9024
	1	36.3576	26.4392	9.9185		
	2	18.1898	13.2246	4.9652		
	3	9.1080	6.6203	2.4877		
	4	4.5643	3.3169	1.2474		
	5	2.2892	1.6633	0.6260		
3	0	52.9024	38.4816	14.4208	0.00000	38.4816
	1	26.4392	19.2277	7.2115		
	2	13.2246	9.6153	3.6093		
	3	6.6203	4.8124	1.8079		
	4	3.3169	2.4106	0.9063		
	5	1.6633	1.2085	0.4548		
4	0	38.4816	27.9936	10.4880	0.00000	27.9936
	1	19.2277	13.9840	5.2436		
	2	9.6153	6.9915	2.6238		
	3	4.8124	3.4984	1.3140		
	4	2.4106	1.7520	0.6586		
	5	1.2085	0.8781	0.3304		
5	0	27.9936	20.3653	7.6283	0.00004	20.3653
	1	13.9840	10.1711	3.8130		
	2	6.9915	5.0840	1.9075		
	3	3.4984	2.5434	0.9551		
	4	1.7520	1.2734	0.4786		
	5	0.8781	0.6381	0.2400		

the tails in the bank are 20.3653
the total concentrate is 79.63468
the total holdup is 212.47478.

SINGLE BANK WITH 5 CELLS

UNIFORM DISTRIBUTION $K1 = 0.49$ $K2 = 0.51$ $h=1$; $l=3$; $T=1$

NO	MOMENT INDEX	FEED(Z)	HOLDUP (HEW0)	CONCENTRATE(ETA)	ERROR	EXACT HOLDUP
1	0	100.0000	72.7280	27.2720	0.00001	72.7280
	1	50.0000	36.3627	13.6373		
	2	25.0033	18.1831	6.8202		
	3	12.5050	9.0936	3.4114		
	4	6.2550	4.5485	1.7065		
	5	3.1292	2.2754	0.8538		
2	0	72.7280	52.8941	19.8339	0.00000	52.8941
	1	36.3627	26.4451	9.9175		
	2	18.1831	13.2234	4.9597		
	3	9.0936	6.6130	2.4207		
	4	4.5485	3.3076	1.2409		
	5	2.2754	1.6546	0.6208		
3	0	52.8941	38.4696	14.4245	0.00000	38.4696
	1	26.4451	19.2327	7.2124		
	2	13.2234	9.6166	3.6068		
	3	6.6130	4.8091	1.8039		
	4	3.3076	2.4052	0.9023		
	5	1.6546	1.2031	0.4514		
4	0	38.4696	27.9790	10.4906	-0.00001	27.9790
	1	19.2327	13.9875	5.2452		
	2	9.6166	6.9936	2.6229		
	3	4.8091	3.4973	1.3118		
	4	2.4052	1.7491	0.6562		
	5	1.2031	0.8749	0.3282		
5	0	27.9790	20.3494	7.6296	0.00000	20.3494
	1	13.9875	10.1728	3.8146		
	2	6.9936	5.0862	1.9075		
	3	3.4973	2.5433	0.9539		
	4	1.7491	1.2719	0.4771		
	5	0.8749	0.6362	0.2387		

the tails in the bank are 20.3494
the total concentrate is 79.65061
the total holdup is 212.42016

DATE 30/10/90

SINGLE BANK WITH 5 CELLS

BETA DISTRIBUTION CAMMA= 1 ETA= 1

h=1 ; l=3 ; T=0.1

NO	MOMENT INDEX	FEED(Z)	HOLDUP (MEW0)	CONCENTRATE(ETA)	ERROR	EXACT HOLDUP	HOLDUP BY ACTUAL INTEGRATION
1	0	100.0000	203.8109	71.6189	0.00009	205.4384	205.34
	1	50.0000	95.4519	40.4509			
	2	33.3333	53.9344	27.9399			
	3	25.0000	37.2532	21.2747			
	4	20.0000	28.3662	17.1634			
	5	16.6667	22.8845	14.3782			
2	0	29.3811	113.3222	17.0489	0.00011	117.8417	117.64
	1	9.5492	22.7318	7.2760			
	2	5.3934	9.7013	4.4233			
	3	3.7253	5.9977	3.1355			
	4	2.9366	4.1807	2.4186			
	5	2.2885	3.2247	1.9660			
3	0	11.3322	61.0360	5.2286	0.05921	66.0360	65.74
	1	2.2732	6.9715	1.5760			
	2	0.9701	2.1014	0.7600			
	3	0.5398	1.0133	0.4884			
	4	0.4181	0.6513	0.3529			
	5	0.3225	0.4766	0.2754			
4	0	6.1036	40.2045	2.0930	0.00000	44.7610	44.37
	1	0.6971	2.7773	0.4194			
	2	0.2101	0.5592	0.1542			
	3	0.1013	0.3056	0.0809			
	4	0.0651	0.1677	0.0544			
	5	0.0471	0.0725	0.0399			
5	0	4.0207	29.7293	1.0478	0.00000	33.8123	33.32
	1	0.2777	1.3971	0.1350			
	2	0.0559	0.1840	0.0375			
	3	0.0206	0.0500	0.0156			
	4	0.0108	0.0207	0.0097			
	5	0.0072	0.0116	0.0061			

the tails in the bank are 2.9729
the total concentrate is 97.02717
the total holdup is 528.10390

SINGLE BANK WITH 5 CELLS

BETA DISTRIBUTION GAMMA= 1 ETA= 2

h=1 ; l=3 ; T=0.1

NO	MOMENT INDEX	FEED(Z)	HOLDUP (MEW0)	CONCENTRATE(ETA)	ERROR	EXACT HOLDUP	HOLDUP BY ACTUAL INTEGRATION
1	0	100.0000	378.7505	62.1249	0.00001	380.3270	380.11
	1	33.3333	82.8333	25.0500			
	2	16.6667	33.4000	13.3267			
	3	10.0000	17.7689	8.2271			
	4	6.6667	10.9641	5.5703			
	5	4.7619	7.4270	4.0192			
2	0	37.8751	186.4680	19.2283	0.00001	190.9910	190.57
	1	8.2833	25.6377	5.7196			
	2	3.3400	7.6261	2.5774			
	3	1.7769	3.4365	1.4332			
	4	1.0964	1.9110	0.9053			
	5	0.7427	1.2071	0.6220			
3	0	18.6468	113.2572	7.3211	0.02066	118.2572	117.65
	1	2.5638	9.7614	1.5876			
	2	0.7626	2.1168	0.5509			
	3	0.3437	0.7346	0.2702			
	4	0.1911	0.3603	0.1551			
	5	0.1207	0.2068	0.1000			
4	0	11.3257	79.8181	3.3439	0.00000	83.8486	83.04
	1	0.9761	4.4585	0.5303			
	2	0.2117	0.7071	0.1410			
	3	0.0735	0.1880	0.0547			
	4	0.0360	0.0729	0.0287			
	5	0.0207	0.0383	0.0168			
5	0	7.9818	62.0669	1.7751	0.00000	64.7049	63.71
	1	0.4459	2.3668	0.2092			
	2	0.0707	0.2789	0.0428			
	3	0.0188	0.0571	0.0131			
	4	0.0073	0.0175	0.0055			
	5	0.0038	0.0074	0.0031			

the tails in the bank are 6.2067
the total concentrate is 93.79331
the total holdup is 920.36070

SINGLE BANK WITH 5 CELLS

BETA DISTRIBUTION GAMMA= 1 ETA= 3

h=1 ; l=3 ; T=0.1

NO	MOMENT INDEX	FEED(Z)	HOLDUP (MEW0)	CONCENTRATE(ETA)	ERROR	EXACT HOLDUP	HOLDUP BY ACTUAL INTEGRATION
1	0	100.0000	445.1529	55.4847	0.00000	446.5559	446.19
	1	25.0000	73.9794	17.6020			
	2	10.0000	23.4694	7.6531			
	3	5.0000	10.2041	3.3796			
	4	2.9571	5.3061	2.3255			
	5	1.7957	3.1020	1.4755			
2	0	44.5153	244.6798	20.0474	0.00000	248.6193	247.96
	1	7.3980	26.7299	4.7250			
	2	2.3469	6.3000	1.7169			
	3	1.0204	2.2893	0.7915			
	4	0.5306	1.0553	0.4251			
	5	0.3102	0.5668	0.2535			
3	0	24.4679	157.8390	8.6840	0.00915	162.8390	161.89
	1	2.6730	11.5706	1.5151			
	2	0.6300	2.0202	0.4288			
	3	0.2289	0.5706	0.1719			
	4	0.1055	0.2291	0.0826			
	5	0.0567	0.1102	0.0457			
4	0	15.7839	113.8913	4.3948	0.00006	118.8913	117.65
	1	1.1579	5.8597	0.5719			
	2	0.2020	0.7625	0.1258			

3	0.0571	0.1677	0.0403				
4	0.0229	0.0537	0.0175				
5	0.0110	0.0234	0.0087				
<hr/>							
5	0	11.3891	98.1906	1.5701	0.00000	93.2286	91.71
1	0.5860	2.0934	0.3766				
2	0.0763	0.5022	0.0260				
3	0.0168	0.0347	0.0133				
4	0.0054	0.0177	0.0036				
5	0.0023	0.0048	0.0019				

the tails in the bank are 9.8191
the total concentrate is 90.18094
the total holdup is 1059.75260

SINGLE BANK WITH 5 CELLS

BETA DISTRIBUTION GAMMA=1 ETA=4

h=1, l=3, T=0.1

NO	MOMENT INDE	FEED(I)	HOLDUP (HEW0)	CONCENTRATE(ETA)	ERROR	EXACT HOLDUP	HOLDUP BY ACTUAL INTEGRATION
<hr/>							
1	0	100.0000	495.6949	50.4305	0.00000	497.0039	496.45
	1	23.0000	67.2407	13.2759			
	2	6.6667	17.7012	4.8965			
	3	2.8571	6.5237	2.2043			
	4	1.4286	2.9390	1.1347			
	5	0.7937	1.5129	0.6424			
<hr/>							
2	0	49.5535	292.8516	20.2843	0.00000	296.3036	295.37
	1	9.7241	27.0453	4.0195			
	2	1.7701	5.3593	1.2342			
	3	0.6539	1.6456	0.4633			
	4	0.2939	0.6511	0.2238			
	5	0.1513	0.3051	0.1208			
<hr/>							
3	0	29.2852	196.9688	9.5183	0.00418	201.9688	200.55
	1	2.7046	12.7977	1.4248			
	2	0.5359	1.8997	0.3460			
	3	0.1646	0.4613	0.1154			
	4	0.0651	0.1579	0.0493			
	5	0.0305	0.0659	0.0239			
<hr/>							
4	0	19.6267	155.7098	4.1169	0.00108	150.7088	151.19
	1	1.2739	5.4880	0.7310			
	2	0.1900	0.9746	0.0925			
	3	0.0461	0.1233	0.0332			
	4	0.0152	0.0451	0.0113			
	5	0.0066	0.0150	0.0051			
<hr/>							
5	0	15.5709	124.7425	3.0966	0.00277	119.7425	117.65
	1	0.5438	4.1288	0.1359			
	2	0.0975	0.1812	0.0793			
	3	0.0123	0.1058	0.0018			
	4	0.0045	0.0023	0.0043			
	5	0.0015	0.0057	0.0009			

the tails in the bank are 12.4743
the total concentrate is 87.52575
the total holdup is 1265.86660

SINGLE BANK WITH 5 CELLS

BETA DISTRIBUTION GAMMA=2 ETA=1

h=1 ; l=3 , T=0.1

NO	MOMENT INDEX	FEED(Z)	HOLDUP (MEUO)	CONCENTRATE(ETA)	ERROR	EXACT HOLDUP	HOLDUP BY ACTUAL INTEGRATION
1	0	100.0000	190.4408	80.9559	0.00008	190.5498	190.58
	1	66.6667	107.9412	55.8725			
	2	50.0000	74.4967	42.5503			
	3	40.0000	56.7338	34.3266			
	4	33.3333	45.7688	28.7565			
	5	29.5714	38.3419	24.7372			
2	0	19.0441	44.3579	14.6083	0.00072	44.6924	44.71
	1	10.7941	19.4777	8.8464			
	2	7.4497	11.7951	6.2702			
	3	5.6734	8.3602	4.8374			
	4	4.5769	6.4498	3.9319			
	5	3.8342	5.2425	3.3099			
3	0	4.4353	13.2729	3.1085	0.00003	13.8149	13.84
	1	1.9478	4.1447	1.5333			
	2	1.1715	2.0444	0.9751			
	3	0.8360	1.3001	0.7060			
	4	0.6450	0.7413	0.5508			
	5	0.5243	0.7345	0.4509			
4	0	1.3273	5.0078	0.8265	0.00006	5.6733	5.70
	1	0.4145	1.1020	0.3043			
	2	0.2044	0.4057	0.1639			
	3	0.1300	0.2185	0.1082			
	4	0.0941	0.1442	0.0797			
	5	0.0734	0.1063	0.0628			
5	0	0.5608	2.2190	0.2739	0.00003	2.9197	2.95
	1	0.1192	0.3718	0.0720			
	2	0.0406	0.0974	0.0303			
	3	0.0218	0.0411	0.0177			
	4	0.0144	0.0237	0.0121			
	5	0.0106	0.0161	0.0090			

the tails in the bank are 0.2219
the total concentrate is 99.77810
the total holdup is 255.29840

SINGLE BANK WITH 5 CELLS

BETA DISTRIBUTION GAMMA=3 ETA=1

h=1 ; l=3 , T=0.1

NO	MOMENT INDEX	FEED(Z)	HOLDUP (MEUO)	CONCENTRATE(ETA)	ERROR	EXACT HOLDUP	HOLDUP BY ACTUAL INTEGRATION
1	0	100.0000	165.1206	83.4879	0.00000	161.8901	161.88
	1	75.0000	111.3173	63.8683			
	2	60.0000	85.1577	51.4842			
	3	50.0000	68.6456	43.1354			
	4	42.8571	57.5139	37.1058			
	5	37.5000	49.4743	32.5526			
2	0	16.5121	28.6105	13.6510	0.22835	29.1715	29.17
	1	11.1317	18.2013	9.3116			
	2	8.5158	12.4155	7.2742			
	3	6.8646	9.6990	5.8947			
	4	5.7514	7.8596	4.9654			

	5	4 9474	6.6206	4.2854			
3	0	2.8611	5.1515	2.3459	0.00010	6.1755	6.17
	1	1.8201	3.1279	1.5073			
	2	1.2415	2.0098	1.0406			
	3	0.9479	1.3874	0.8312			
	4	0.7860	1.1082	0.6751			
	5	0.6621	0.9002	0.5720			
4	0	0.5152	0.9740	0.4178	0.00002	1.6283	1.62
	1	0.3129	0.5570	0.2571			
	2	0.2010	0.3428	0.1667			
	3	0.1357	0.2223	0.1165			
	4	0.1102	0.1554	0.0953			
	5	0.0900	0.1270	0.0773			
5	0	0.0974	0.6610	0.0373	0.00319	0.5507	0.55
	1	0.0557	0.0497	0.0507			
	2	0.0343	0.0676	0.0275			
	3	0.0222	0.0367	0.0186			
	4	0.0155	0.0247	0.0131			
	5	0.0127	0.0174	0.0110			

the tails in the bank are 0.6601
the total concentrate is 99.93990
the total holdup is 260.45766

SINGLE BANK WITH 5 CELLS

BETA DISTRIBUTION GAMMA = 4 ETA = 1

h=1 ; l=3 ; T=0.1

NO	MOMENT INDEX	FEED (Z)	HOLDUP (HEUO)	CONCENTRATE (ETA)	ERROR	EXACT HOLDUP	HOLDUP BY ACTUAL INTEGRATION
1	0	100.0000	150.2162	84.5790	0.00001	149.0112	148.99
	1	80.0000	113.3353	68.6695			
	2	66.6667	91.5533	57.5107			
	3	57.1429	76.6810	49.4748			
	4	50.0000	65.9663	43.4034			
	5	44.4444	57.8712	38.6573			
2	0	15.0210	23.7424	12.6465	0.12227	23.5944	23.59
	1	11.3305	16.5624	9.6443			
	2	9.1559	12.5591	7.6700			
	3	7.6681	10.4934	6.6128			
	4	6.5966	8.8250	5.7141			
	5	5.7671	7.6188	5.0252			
3	0	2.3742	3.8310	1.9911	0.00000	4.0882	4.09
	1	1.6862	2.6549	1.4208			
	2	1.2850	1.9943	1.0965			
	3	1.0493	1.4620	0.9031			
	4	0.8525	1.2042	0.7621			
	5	0.7619	1.0161	0.6603			
4	0	0.3831	0.5790	0.3252	0.00002	0.8084	0.81
	1	0.2655	0.4336	0.2221			
	2	0.1894	0.2962	0.1598			
	3	0.1462	0.2131	0.1249			
	4	0.1264	0.1665	0.1038			
	5	0.1016	0.1384	0.0878			
5	0	0.0579	0.5000	0.0079	0.00387	0.1916	0.19
	1	0.0434	0.0105	0.0423			
	2	0.0296	0.0564	0.0240			
	3	0.0213	0.0320	0.0181			
	4	0.0167	0.0241	0.0142			
	5	0.0138	0.0190	0.0119			

the tails in the bank are 0.0579
the total concentrate is 99.94210
the total holdup is 178.36260

SINGLE BANK WITH 5 CELLS

BETA DISTRIBUTION GAMMA= 2 ETA= 3

h=1 ; l=3 ; T=0

NO	MOMENT INDEX	FEED(Z)	HOLDUP (HEU0)	CONCENTRATE(ETA)	ERROR	EXACT HOLDUP	HOLDUP BY ACTUAL INTEGRATION
1	0	100.0000	295.1754	70.4825	0.00001	295.2119	294.15
	1	40.0000	93.9766	30.6023			
	2	20.0000	40.8031	15.9197			
	3	11.4226	21.2263	9.3059			
	4	7.1429	12.4079	5.9021			
	5	4.7619	7.2694	3.9750			
2	0	29.5175	105.1587	19.0017	0.00001	105.5662	105.57
	1	9.3977	25.7356	6.8641			
	2	4.0303	9.1521	3.1651			
	3	2.1526	4.2201	1.7006			
	4	1.2468	2.2675	1.0140			
	5	0.7119	1.3521	0.5517			
3	0	10.5159	44.5210	0.5236	0.00001	45.7495	45.90
	1	2.8736	5.3217	1.7304			
	2	0.9152	2.3072	0.6845			
	3	0.4220	0.9127	0.3307			
	4	0.2207	0.4410	0.1826			
	5	0.1352	0.2435	0.1109			
4	0	4.4921	20.2153	1.6706	0.00000	23.4388	23.59
	1	0.8032	2.2274	0.5804			
	2	0.2307	0.7739	0.1533			
	3	0.1313	0.3044	0.0708			
	4	0.0441	0.0944	0.0347			
	5	0.0244	0.0462	0.0197			
5	0	2.8215	16.0328	1.0187	0.00435	13.6868	13.84
	1	0.2327	1.3562	0.0869			
	2	0.0774	0.1159	0.0658			
	3	0.0204	0.0877	0.0117			
	4	0.0094	0.0156	0.0079			
	5	0.0046	0.0105	0.0036			

the tails in the bank are 1.8029
the total concentrate is 98.19712
the total holdup is 491.49920

SINGLE BANK WITH 5 CELLS

BETA DISTRIBUTION GAMMA= 3 ETA= 2

h=1 ; l=3 ; T=0.1

NO	MOMENT INDEX	FEED(Z)	HOLDUP (MEU0)	CONCENTRATE(ETA)	ERROR	EXACT HOLDUP	HOLDUP BY ACTUAL INTEGRATION
1	0	100.0000	200.5185	79.9431			
	1	60.0000	106.5975	49.3402	0.00004	200.5265	200.50
	2	40.0000	65.7870	33.4213			
	3	28.5714	44.5617	24.1153			
	4	21.4286	32.1537	18.2132			
	5	16.6667	24.2843	14.2382			
2	0	20.0519	49.1166	15.1402	0.00000	45.9026	45.90
	1	10.6598	20.1869	8.6411			
	2	6.5787	11.5214	5.4266			
	3	4.4562	7.2354	3.7326			
	4	3.2154	4.9768	2.7177			
	5	2.4284	3.6236	2.0661			
3	0	4.9117	12.1656	3.6951	0.05606	12.4376	12.44
	1	2.0187	4.9268	1.5260			
	2	1.1521	2.0347	0.9487			
	3	0.7235	1.2649	0.5971			
	4	0.4977	0.7961	0.4181			
	5	0.3624	0.5574	0.3066			
4	0	1.2166	3.0310	0.9135	0.00030	4.0880	4.09
	1	0.4927	1.2179	0.3709			
	2	0.2035	0.4945	0.1540			
	3	0.1265	0.2054	0.1060			
	4	0.0776	0.1413	0.0655			
	5	0.0557	0.0873	0.0470			
5	0	0.3031	0.7650	0.2266	0.00006	1.6282	1.63
	1	0.1218	0.3021	0.0716			
	2	0.0495	0.1221	0.0372			
	3	0.0205	0.0497	0.0156			
	4	0.0141	0.0208	0.0121			
	5	0.0087	0.0161	0.0071			

the tails in the bank are 0.0765
the total concentrate is 97.92350
the total holdup is 265.59670

DATE 30/10/90

SINGLE BANK WITH 5 CELLS

MIXTURE OF TWO BETA DISTRIBUTIONS IN PROPORTIONS m_1 m_2 CAMMA1= 2, ETA1= 3, m_1 = 0.10CAMMA2= 3, ETA2= 2, m_2 = 0.90

h=1 ; l=3 ; T=0.1

NO	MOMENT INDEX	FFED(Z)	HOLDUP (MEU0)	CONCENTRATE(ETA)	ERROR	EXACT HOLDUP	HOLDUP BY ACTUAL INTEGRATION
1	0	100.0000	210.0251	78.9975	0.00004	209.9951	209.90
	1	58.0000	105.3300	47.4670			
	2	39.0000	63.2893	31.6711			
	3	26.2571	42.2281	22.6343			
	4	20.0000	30.1791	16.9831			
	5	15.4762	22.6429	13.2119			
2	0	21.0025	51.2490	15.8176	0.00071	51.8690	51.87
	1	10.5330	21.0901	8.4240			
	2	6.3239	11.2329	5.2057			
	3	4.2228	6.9410	3.5297			
	4	3.0179	4.7049	2.5474			
	5	2.2643	3.3966	1.9246			
3	0	5.1849	15.6789	3.6170	0.00026	15.7683	15.78
	1	2.1090	4.9227	1.6357			
	2	1.1232	2.1690	0.9063			
	3	0.6941	1.2084	0.5733			
	4	0.4705	0.7643	0.3941			
	5	0.3397	0.5254	0.2971			
4	0	1.5679	8.3931	0.7286	0.00001	8.0231	8.04
	1	0.4823	0.9714	0.3951			
	2	0.2169	0.5135	0.1655			
	3	0.1208	0.2207	0.0953			
	4	0.0764	0.1317	0.0632			
	5	0.0525	0.0844	0.0441			
5	0	0.5393	2.3900	0.5503	0.00484	2.8341	2.85
	1	0.0971	0.7337	0.0239			
	2	0.0514	0.0317	0.0482			
	3	0.0321	0.0642	0.0157			
	4	0.0132	0.0209	0.0111			
	5	0.0094	0.0148	0.0070			

the tails in the bank are 0.2890
the total concentrate is 99.71100
the total holdup is 289.83600

SINGLE BANK WITH 5 CELLS

MIXTURE OF TWO BETA DISTRIBUTIONS IN PROPORTIONS $m_1:m_2$ GAMMA1= 2, ETA1= 3, $m_1= 0.20$ GAMMA2= 3, ETA2= 2, $m_2= 0.80$

h=1 ; l=3 ; T=0.1

NO	MOMENT INDEX	FEED(I)	HOLDUP (MEU0)	CONCENTRATE(ETA)	ERROR	EXACT HOLDUP	HOLDUP BY ACTUAL INTEGRATION
1	0	100.0000	219.5236	78.0476	0.00021	219.4636	219.23
	1	56.0000	104.0635	45.5936			
	2	36.0000	60.7915	29.9208			
	3	25.1429	39.8945	21.1534			
	4	18.5714	28.2045	15.7510			
	5	14.2857	21.0013	12.1856			
2	0	21.9524	57.8353	16.1488	0.00035	57.8353	57.84
	1	10.4064	21.5584	8.2505			
	2	6.0792	11.0007	4.9791			
	3	3.9894	6.6388	3.3256			
	4	2.8205	4.4341	2.3770			
	5	2.1001	3.1694	1.7832			
3	0	5.7835	18.9900	3.8845	0.00034	19.1000	19.13
	1	2.1558	5.1794	1.6379			
	2	1.1001	2.1839	0.8817			
	3	0.6639	1.1756	0.5463			
	4	0.4434	0.7294	0.3706			
	5	0.3165	0.4941	0.2675			
4	0	1.8990	7.7282	1.1263	0.00004	7.9532	7.99
	1	0.5179	1.5016	0.3679			
	2	0.2184	0.4904	0.1694			
	3	0.1176	0.3258	0.0950			
	4	0.0729	0.1266	0.0602			
	5	0.0494	0.0802	0.0414			
5	0	0.7729	5.4399	0.2293	0.00000	4.0399	4.07
	1	0.1502	0.3051	0.1196			
	2	0.0495	0.1575	0.0331			
	3	0.0324	0.0441	0.0192			
	4	0.0127	0.0242	0.0102			
	5	0.0090	0.0137	0.0067			

the tails in the bank are 0.5440.
the total concentrate is 79.45601
the total holdup is 309.51700

SINGLE BANK WITH 5 CELLS

MIXTURE OF TWO BETA DISTRIBUTIONS IN PROPORTIONS $m_1:m_2$ GAMMA1= 2, ETA1= 3, $m_1= 0.25$ GAMMA2= 3, ETA2= 2, $m_2= 0.75$

h=1 ; l=3 ; T=0.1

NO	MOMENT INDEX	FEED(I)	HOLDUP (MEU0)	CONCENTRATE(ETA)	ERROR	EXACT HOLDUP	HOLDUP BY ACTUAL INTEGRATION
1	0	100.0000	224.2479	77.5752	0.00043	224.1979	223.91
	1	55.0000	103.4336	44.6566			
	2	35.0000	59.5422	29.0459			
	3	24.2957	38.7277	20.4129			
	4	17.8571	27.2173	15.1354			
	5	13.6905	20.1806	11.6724			
2	0	22.4248	60.7785	16.3469	0.00042	60.8185	60.82
	1	10.3434	21.7959	8.1638			
	2	5.9542	10.8850	4.8657			
	3	3.8728	6.4876	3.2240			
	4	2.7217	4.2987	2.2919			

	5	2 0181	3.0558	1 7125			
3	0	6 0779	20 5355	4.0193	0.00002	20.7655	20 80
	1	2 1796	5 3591	1 6437			
	2	1.0885	2.1916	0.8693			
	3	0.6488	1 1591	0.5328			
	4	0 4299	0 7105	0 3588			
	5	0 3056	0.4784	0.2577			
4	0	2 0586	8 6057	1.1980	0.00023	8.9257	8 97
	1	0 5359	1 5973	0 3762			
	2	0 2192	0 5016	0 1690			
	3	0 1159	0 2253	0 0934			
	4	0 0710	0 1245	0.0586			
	5	0 0478	0.0791	0.0400			
5	0	0 8606	6.0529	0.2553	0.00000	4.6429	4 68
	1	0 1597	0 3404	0 1257			
	2	0 0502	0 1676	0 0334			
	3	0 0225	0 0445	0.0191			
	4	0 0125	0 0241	0 0100			
	5	0 0072	0.0134	0 0065			

the tails in the bank are 0 6053
the total concentrate is 99 39471
the total holdup is 320 27050

SINGLE BANK WITH 5 CELLS

MIXTURE OF TWO BETA DISTRIBUTIONS IN PROPORTIONS $m_1:m_2$

GAMMA1= 2, ETA1= 3, $m_1= 0.30$

GAMMA2= 3, ETA2= 2, $m_2= 0.70$

$h=1$; $l=3$; $T=0.1$

NO	MOMENT INDEX	FEED(Z)	HOLDUP (MEW0)	CONCENTRATE(ET+)	ERROR	EXACT HOLDUP	HOLDUP BY ACTUAL INTEGRATION
1	0	100 0000	238 9721	77 1029	0 00077	228 9321	228.60
	1	54 0000	102 8037	43 7136			
	2	34 0000	58 2928	28 1707			
	3	23 4286	37 5610	19 6725			
	4	17 1429	26 3300	14 5199			
	5	13 0752	19 3599	11 1593			
2	0	22 2972	63 7117	16 5260	0.00047	63 8017	63.84
	1	10 2804	22 0347	8 0769			
	2	5 8293	10 7632	4 7524			
	3	3 7561	6 3365	3 1224			
	4	2.6230	4 1633	2 2067			
	5	1 9360	2 9422	1 6418			
3	0	6 3712	22 1611	4.1551	0.00012	22.4311	22.48
	1	2.2035	5 5401	1.6495			
	2	1 0769	2 1993	0 8570			
	3	0 6336	1 1427	0 5194			
	4	0.4163	0.6925	0 3471			
	5	0 2942	0.4628	0.2479			
4	0	2.2161	9 4533	1.2708	0.00005	9.8933	9.94
	1	0 5540	1 6944	0.3846			
	2	0.2199	0.5128	0.1687			
	3	0 1143	0.2249	0.0918			
	4	0.0693	0.1224	0.0570			

S	0.0463	0.0760	0.0397			
5	0	0.9453	6.5755	0.2878	0.00000	5.2458
1	0	0.1694	0.3837	0.1311		5.29
2	0	0.0513	0.1748	0.0339		
3	0	0.0225	0.0451	0.0180		
4	0	0.0122	0.0240	0.0098		
5	0	0.0076	0.0131	0.0063		

the tails in the bank are 0.6576
the total concentrate is 99.34242
the total holdup is 330.87400

SINGLE BANK WITH 5 CELLS

MIXTURE OF TWO BETA DISTRIBUTIONS IN PROPORTIONS $m_1:m_2$ GAMMA1= 2, ETA1= 3, $m_1= 0.40$ GAMMA2= 3, ETA2= 3, $m_2= 0.60$ $h=1$; $l=3$; $T=1$

NO	MOMENT INDEX	FEED(Z)	HOLDUP (MEQ)	CONCENTRATE(ETA)	ERROR	EXACT HOLDUP	HOLDUP BY ACTUAL INTEGRATION
1	0	100.0000	239.3707	76.1629	0.00065	239.4007	237.96
	1	52.0000	101.5503	41.8440			
	2	33.0000	55.1925	26.4207			
	3	21.7143	35.2276	19.1915			
	4	15.7143	24.2554	13.3187			
	5	11.9943	17.7193	10.1229			
2	0	27.9371	69.4980	16.2573	0.00037	69.7680	69.77
	1	10.1551	28.5164	7.8034			
	2	5.5793	13.5379	4.5355			
	3	3.8219	8.7340	2.8194			
	4	2.4855	5.4925	1.9363			
	5	1.7713	3.7151	1.5003			
3	0	19.499	35.2227	4.4276	0.00004	35.7623	35.82
	1	8.2516	14.4034	1.8617			
	2	4.0533	7.1151	0.9323			
	3	2.6024	4.1097	0.4924			
	4	1.3792	2.3566	0.2336			
	5	0.8718	1.4315	0.1284			
4	0	11.9232	11.0533	1.1124	0.00003	11.8283	11.99
	1	6.5903	5.8372	0.4016			
	2	3.2115	2.9355	0.1680			
	3	1.1110	0.8239	0.0836			
	4	0.6657	0.4181	0.0532			
	5	0.4431	0.0713	0.0360			
5	0	1.1068	7.4217	0.3647	0.00000	6.4517	6.51
	1	0.1887	0.4202	0.1401			
	2	0.0535	0.1868	0.0349			
	3	0.0224	0.0465	0.0177			
	4	0.0118	0.0237	0.0094			
	5	0.0072	0.0126	0.0059			

the tails in the bank are 0.7422
the total concentrate is 99.25783
the total holdup is 351.58100

